Chapter V

Analysis of Scattered Waves from Rough Burnt Coal Seam

5.1 Introduction

A simple analysis to solve scattered waves from burnt coal seam using the classical transmission line method was discussed in Chapter IV, where the roughness of targeted surface is ignored (Tetuko *et al*. 2001). In this Chapter, the author discusses more complicated scattered wave analysis from burnt coal seam by considering the roughness of it. Figure 4.2 (b) to (d) show distribution of the coal seam thickness in the study area: ‘One Million Hectares Peatland Project’ District A (PLG-A), B (PLG-B) and D (PLG-D) at central Borneo, Indonesia (CSAR 1997). The ground data was collected from 1995 to 1997. The acquired data shows that the burnt coal seam surface in the study area had standard deviation d of roughness about 0.3m (Nuraini 1999). It means d is larger than wavelength of Japanese Earth Resources Satellite (JERS-1) Synthetic Aperture Radar (SAR) that was operating at frequency 1.275 GHz (L Band). Thus, in this study, additional simplifying assumption using the stationary-phase approximation (Ulaby *et al*. 1986) is needed. In this study, the method is advanced to obtain analytical solutions of scattered wave from two layers of rough surfaces.

In section 5.2, the analysis of scattered waves from the rough burnt coal seam will be discussed. The confirmation of the analysis results by comparing its with results of previous simple analysis is discussed in section 5.3. Section 5.4 shows the application of the proposed analysis results to estimate the thickness of burnt coal seam in the study area, 'One Million Hectares Peatland Project' (PLG) using JERS-1 SAR data. Finally, conclusions are shown in section 5.5.

5.2 Analysis

The Kirchhoff or Physical Optics formulation is applicable to surfaces with gentle undulations where average horizontal dimension is large compared with the incident wave (Ulaby *et al*. 1986). As a result, it is assumed that the total field at any point on the surface can be computed as if the incident wave is impinging upon an infinite plane tangent to the point. The vector formulation of the Kirchhoff method is based upon the vector second Green's theorem, which states that the scattered field at any point within a source-free region bounded by a closed surface can be expressed in terms of the tangential fields on the surface (Fung *et al*. 1992). A mathematical statement of this fact formulated by Stratton and Chu (Stratton 1941) and modified for the far zone by Silver (Silver 1947) is shown by

$$\boldsymbol{E}^{s} = K\hat{\boldsymbol{n}}_s \times \int \left(\hat{\boldsymbol{n}} \times \boldsymbol{E} - h\hat{\boldsymbol{n}}_s \times \left(\hat{\boldsymbol{n}} \times \boldsymbol{H}\right)\right) e^{jk_s \boldsymbol{r} \cdot \hat{\boldsymbol{n}}_s} dS' \tag{5.1}$$

where a time factor of the form e^{jwt} is understood and $K = -jk_s e^{-jk_s R_o} / (4pR)$. $\hat{\boldsymbol{n}}_s$, $\hat{\boldsymbol{n}}$, h, k_s, R, $\boldsymbol{E}$, and $\boldsymbol{H}$ are unit vector in the scattered direction, unit vector normal to interface inside the medium in which scattering is considered, intrinsic impedance of the medium in

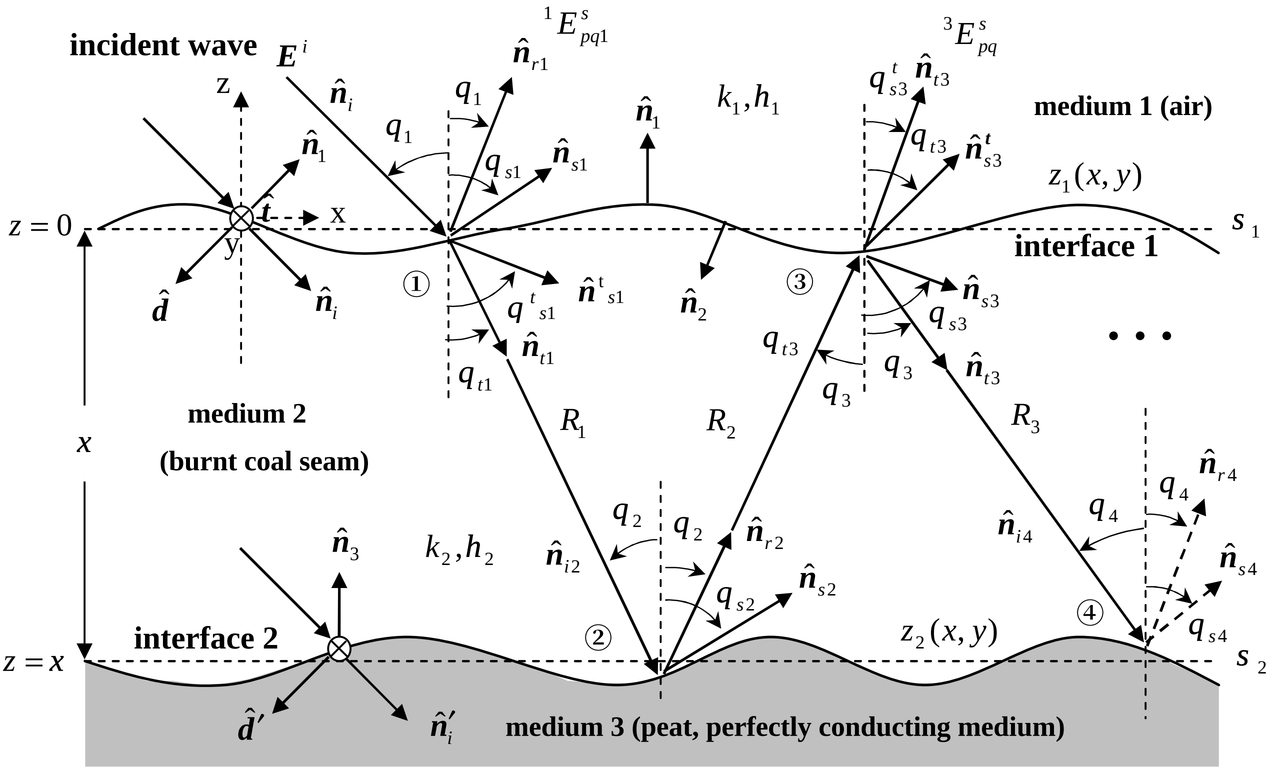

Figure 5.1. Geometry of the scattered waves analysis.

which $\boldsymbol{E}^s$ is evaluated, wavenumber of the medium in which $\boldsymbol{E}^s$ is evaluated, range from the centre of the illuminated area to the point of observation, total electric and magnetic fields on the interface, respectively. In a local frame of reference, the tangential fields $\hat{\boldsymbol{n}}\times\boldsymbol{E}$ and $\hat{\boldsymbol{n}}\times\boldsymbol{H}$ are calculated to compute the scattered field $\boldsymbol{E}^s$. The incident wave is assumed to be (see figure 5.1)

$$\boldsymbol{E}^i = \hat{\boldsymbol{a}} E_o \exp\left(-jk_1 \hat{\boldsymbol{n}}_i \cdot \boldsymbol{r}\right) \tag{5.2}$$

where $\hat{\boldsymbol{a}}$, $\hat{\boldsymbol{n}}_i$, and k_1 are unit polarization vector, unit vector in the incident direction, and wavenumber in medium 1 (air), respectively. Unit vectors in a local frame of reference are

$$\hat{\boldsymbol{t}} = \hat{\boldsymbol{n}}_i \times \hat{\boldsymbol{n}}_1 \big/ \left| \hat{\boldsymbol{n}}_i \times \hat{\boldsymbol{n}}_1 \right| \tag{5.3}$$

$$\hat{\boldsymbol{d}} = \hat{\boldsymbol{n}}_i \times \hat{\boldsymbol{t}} \tag{5.4}$$

$$\hat{\boldsymbol{n}}_i = \hat{\boldsymbol{t}} \times \hat{\boldsymbol{d}} \tag{5.5}$$

where $\hat{\boldsymbol{n}}_1$ is unit normal vector to the interface in medium 1. Horizontally and vertically components of incident wave amplitude are locally resolved as

$$\boldsymbol{E}_{\perp}^{i} = \left(\hat{\boldsymbol{a}} \cdot \hat{\boldsymbol{t}}\right) \hat{\boldsymbol{t}} E_{\boldsymbol{o}} \tag{5.6}$$

$$\boldsymbol{H}_{\perp}^{i} = \hat{\boldsymbol{n}}_i \times \left(\left(\hat{\boldsymbol{a}} \cdot \hat{\boldsymbol{t}}\right) \hat{\boldsymbol{t}} E_o\right) \big/ h_1 = \left(\hat{\boldsymbol{a}} \cdot \hat{\boldsymbol{t}}\right) \hat{\boldsymbol{d}} E_o \big/ h_1 \tag{5.7}$$

$$\boldsymbol{E}_{\parallel}^{i} = \left(\hat{\boldsymbol{a}} \cdot \hat{\boldsymbol{d}}\right) \hat{\boldsymbol{d}} E_o \tag{5.8}$$

$$\boldsymbol{H}_{\parallel}^{i} = \hat{\boldsymbol{n}}_i \times \left(\left(\hat{\boldsymbol{a}} \cdot \hat{\boldsymbol{d}}\right) \hat{\boldsymbol{d}} E_o\right) \big/ h_1 = -\left(\hat{\boldsymbol{a}} \cdot \hat{\boldsymbol{d}}\right) \hat{\boldsymbol{t}} E_o \big/ h_1 \tag{5.9}$$

where h_1 is the intrinsic impedance of medium 1. Under the tangent-plane approximation, the total field at a point on the surface is equal to the incident field plus the field reflected by an infinite plane tangent to the point. Thus, the tangential horizontally polarized fields are

$$\hat{\boldsymbol{n}}_1 \times \boldsymbol{E}_{\perp} = \hat{\boldsymbol{n}}_1 \times \left(\boldsymbol{E}_{\perp}^{i} + \boldsymbol{E}_{\perp}^{r}\right) = \hat{\boldsymbol{n}}_1 \times \boldsymbol{E}_{\perp}^{i} \left(1 + R_{\perp}\right) \tag{5.10}$$

$$\begin{aligned} \hat{\boldsymbol{n}}_1 \times \boldsymbol{H}_{\perp} &= \hat{\boldsymbol{n}}_1 \times \left(\boldsymbol{H}_{\perp}^{i} + \boldsymbol{H}_{\perp}^{r}\right) = \hat{\boldsymbol{n}}_1 \times \left(\hat{\boldsymbol{n}}_i \times \boldsymbol{E}_{\perp}^{i} + \hat{\boldsymbol{n}}_r \times \boldsymbol{E}_{\perp}^{i} R_{\perp}\right) \big/ h_1 \\ &= -\left[\left(\hat{\boldsymbol{n}}_1 \cdot \hat{\boldsymbol{n}}_i\right) \boldsymbol{E}_{\perp}^{i} + \left(\hat{\boldsymbol{n}}_1 \cdot \hat{\boldsymbol{n}}_r\right) \boldsymbol{E}_{\perp}^{i} R_{\perp}\right] \big/ h_1 \\ &= -\left(\hat{\boldsymbol{n}}_1 \cdot \hat{\boldsymbol{n}}_i\right)\left(1 - R_{\perp}\right) \boldsymbol{E}_{\perp}^{i} \big/ h_1 \end{aligned} \tag{5.11}$$

where $\hat{\boldsymbol{n}}_r$ is the unit vector in the reflected field direction and $R_{\perp}$ is the Fresnel reflection coefficient for horizontal polarization. In a similar way, the tangential vertically polarized fields can also be found:

$$\hat{\boldsymbol{n}}_{\boldsymbol{1}} \times \boldsymbol{H}_{\parallel} = \hat{\boldsymbol{n}}_{\boldsymbol{1}} \times \left(\boldsymbol{H}_{\parallel}^{i} + \boldsymbol{H}_{\parallel}^{r}\right) = \hat{\boldsymbol{n}}_1 \times \boldsymbol{H}_{\parallel}^{i} \left(1 + R_{\parallel}\right) \tag{5.12}$$

$$\hat{\boldsymbol{n}}_{\boldsymbol{1}} \times \boldsymbol{E}_{\parallel} = \hat{\boldsymbol{n}}_{\boldsymbol{1}} \times \left(\boldsymbol{E}_{\parallel}^{i} + \boldsymbol{E}_{\parallel}^{r}\right) = h_1 \hat{\boldsymbol{n}}_{\boldsymbol{1}} \times \left(\hat{\boldsymbol{n}}_{\boldsymbol{i}} \times \boldsymbol{H}_{\parallel}^{i} + \hat{\boldsymbol{n}}_{\boldsymbol{r}} \times \boldsymbol{H}_{\parallel}^{r}\right)$$
$$= h_1 \left(\hat{\boldsymbol{n}}_1 \cdot \hat{\boldsymbol{n}}_i\right) \boldsymbol{H}_{\parallel}^{i} \left(1 - R_{\parallel}\right) \qquad (5.13)$$

where $R_{\parallel}$ is the Fresnel reflection coefficient for vertical polarization. By summing the locally horizontal and vertical electric and magnetic tangential fields, the total electric and magnetic tangential fields are obtained as

$$\hat{\boldsymbol{n}}_1 \times \boldsymbol{E} = \left(\left(1 + R_{\perp}\right)\left(\hat{\boldsymbol{a}} \cdot \hat{\boldsymbol{t}}\right)\left(\hat{\boldsymbol{n}}_1 \times \hat{\boldsymbol{t}}\right) - \left(1 - R_{\parallel}\right)\left(\hat{\boldsymbol{n}}_1 \cdot \hat{\boldsymbol{n}}_i\right)\left(\hat{\boldsymbol{a}} \cdot \hat{\boldsymbol{d}}\right)\hat{\boldsymbol{t}}\right)E_o \qquad (5.14)$$

$$h_{\boldsymbol{1}}\left(\hat{\boldsymbol{n}}_1 \times \boldsymbol{H}\right) = -\left(\left(1 - R_{\perp}\right)\left(\hat{\boldsymbol{n}}_1 \cdot \hat{\boldsymbol{n}}_i\right)\left(\hat{\boldsymbol{a}} \cdot \hat{\boldsymbol{t}}\right)\hat{\boldsymbol{t}} + \left(1 + R_{\parallel}\right)\left(\hat{\boldsymbol{a}} \cdot \hat{\boldsymbol{d}}\right)\left(\hat{\boldsymbol{n}}_1 \times \hat{\boldsymbol{t}}\right)\right)E_o \qquad (5.15)$$

Due to the continuity of the tangential fields at a surface boundary, the scattered field in either medium 1 or medium 2 can be computed in terms of (5.14) and (5.15). In the same way, the tangential fields in terms of medium 2 parameters are obtained as

$$\hat{\boldsymbol{n}}_2 \times \boldsymbol{E} = \left(T_{\perp}\left(\hat{\boldsymbol{n}}_2 \times \hat{\boldsymbol{t}}\right)\left(\hat{\boldsymbol{a}} \cdot \hat{\boldsymbol{t}}\right) - T_{\parallel}\hat{\boldsymbol{t}}\left(\hat{\boldsymbol{n}}_2 \cdot \hat{\boldsymbol{n}}_t\right)\left(\hat{\boldsymbol{a}} \cdot \hat{\boldsymbol{d}}\right)h_2/h_1\right)E_o \qquad (5.16)$$

$$h_{\boldsymbol{1}}\left(\hat{\boldsymbol{n}}_2 \times \boldsymbol{H}\right) = -\left(T_{\perp}\hat{\boldsymbol{t}}\left(\hat{\boldsymbol{n}}_2 \cdot \hat{\boldsymbol{n}}_t\right)\left(\hat{\boldsymbol{a}} \cdot \hat{\boldsymbol{t}}\right) + T_{\parallel}\left(\hat{\boldsymbol{n}}_2 \times \hat{\boldsymbol{t}}\right)\left(\hat{\boldsymbol{a}} \cdot \hat{\boldsymbol{d}}\right)h_2/h_1\right)E_o \qquad (5.17)$$

where $\hat{\boldsymbol{n}}_2$ is the surface normal in medium 2. $\hat{\boldsymbol{n}}_t$ is the unit vector in the transmitted field direction. h_2 is the intrinsic impedance of medium 2, and $T_{\perp}$, $T_{\parallel}$ are the Fresnel transmission coefficient for horizontal and vertical polarizations, respectively, with $T_{\perp} = 1 + R_{\perp}$ and $T_{\parallel} = 1 + R_{\parallel}$. Finally, we obtained (5.14) and (5.15), (5.16) and (5.17) to calculate the scattered fields in medium 1 and 2, respectively.

5.2.1 Scattered fields on burnt coal seam surface (1) or interface 1

5.2.1.1 Scattering field on medium 1

The scattered fields in medium 1 (air) is obtained by substituting (5.14) and (5.15) to (5.1) as

$$^{1}\boldsymbol{E}_1^s = {}^{1}\mathrm{K}_1 \hat{\boldsymbol{n}}_{s1} \times \int \left[\hat{\boldsymbol{n}}_1 \times \boldsymbol{E} - h_1 \hat{\boldsymbol{n}}_{s1} \times \left(\hat{\boldsymbol{n}}_1 \times \boldsymbol{H} \right) \right] \exp\left[jk_1 \left(\hat{\boldsymbol{n}}_{s1} - \hat{\boldsymbol{n}}_i \right) \cdot \boldsymbol{r}' \right] d\boldsymbol{S}' \tag{5.18}$$

where $^{1}\mathrm{K}_1 = -jk_1 \exp(-jk_1 R)/(4pR)$ and R is range from the centre of the illuminated area to the point of observation.

$$\hat{\boldsymbol{n}}_i = \hat{\boldsymbol{x}} \sin q_1 \cos f_1 + \hat{\boldsymbol{y}} \sin q_1 \sin f_1 - \hat{\boldsymbol{z}} \cos q_1 \tag{5.19}$$

$$\hat{\boldsymbol{n}}_{s1} = \hat{\boldsymbol{x}} \sin q_{s1} \cos f_{s1} + \hat{\boldsymbol{y}} \sin q_{s1} \sin f_{s1} + \hat{\boldsymbol{z}} \cos q_{s1} \tag{5.20}$$

Where q_1 and q_{s1} are incident and scattered angle on ① at figure 5.1, respectively. (5.18) shows that surface fields can be summed over the illuminated area with the phase factor, $\exp(-jk_1 \hat{\boldsymbol{n}}_i \cdot \boldsymbol{r}')$, of the incident wave. The next approximation is considered to find the analytical solution of (5.18). It means that scattering can occur only along directions for which there are specular points on the surface. The approximation relations are obtained from the phase Q of (5.18), i.e.,

$$Q = k_1 \left(\hat{\boldsymbol{n}}_{s1} - \hat{\boldsymbol{n}}_i \right) \cdot \hat{\boldsymbol{r}}' \equiv q_{x1} x' + q_{y1} y' + q_{z1} z' \tag{5.21}$$

where

$$q_{x1} = k_1 \left(\sin q_{s1} \cos f_{s1} - \sin q_1 \cos f_1 \right) \tag{5.22}$$

$$q_{y1} = k_1 \left(\sin q_{s1} \sin f_{s1} - \sin q_1 \sin f_1 \right) \tag{5.23}$$

$$q_{z1} = k_1 (\cos q_{s1} - \cos q_1) \tag{5.24}$$

The phase Q is said to be stationary at a point if its rate of change is zero at the point. Hence, the partial derivations of the surface slopes can be replaced by the components of the phase as

$$\frac{\partial z'}{\partial x'} = -\frac{q_{x1}}{q_{z1}} \tag{5.25}$$

$$\frac{\partial z'}{\partial y'} = -\frac{q_{y1}}{q_{z1}} \tag{5.26}$$

By applying (5.25) and (5.26) to (5.18), the expression for ${}^{1}\boldsymbol{E}_1^{S}$ can be rewritten as

$${}^{1}\boldsymbol{E}_1^{S} = {}^{1}\mathrm{K}_1 \hat{\boldsymbol{n}}_{s1} \times (\hat{\boldsymbol{n}}_1 \times \boldsymbol{E} - h_1 \hat{\boldsymbol{n}}_{s1} \times (\hat{\boldsymbol{n}}_1 \times \boldsymbol{H}))\ {}^{1}I_1 \tag{5.27}$$

where

$${}^{1}I_1 = \int \exp(jk_1(\hat{\boldsymbol{n}}_{s1} - \hat{\boldsymbol{n}}_i) \cdot \boldsymbol{r}')\, dS' \tag{5.28}$$

Let $\hat{\boldsymbol{v}}$ and $\hat{\boldsymbol{h}}$ be unit polarization vectors for the incident vertical and horizontal waves respectively. Let $\hat{\boldsymbol{v}}_{s1}$ and $\hat{\boldsymbol{h}}_{s1}$ be the corresponding polarization vectors for the scattered waves that are chosen to coincide with $\hat{q}$ and $\hat{f}$ in a standard spherical coordinate system. Thus

$$\hat{\boldsymbol{h}}_{s1} = \hat{f} = -\hat{\boldsymbol{x}} \sin f_{s1} + \hat{\boldsymbol{y}} \cos f_{s1} \tag{5.29}$$

$$\hat{\boldsymbol{v}}_{s1} = \hat{q} = \hat{\boldsymbol{h}}_{s1} \times \hat{\boldsymbol{n}}_{s1} = \hat{\boldsymbol{x}} \cos q_{s1} \cos f_{s1} + \hat{\boldsymbol{y}} \cos q_{s1} \sin f_{s1} - \hat{\boldsymbol{z}} \sin q_{s1} \tag{5.30}$$

Since $\hat{\boldsymbol{n}}_i$ has a negative z-component, horizontally and vertically polarised unit vector are

$$\hat{\boldsymbol{h}}_1 = -\hat{\boldsymbol{x}} \sin f_1 + \hat{\boldsymbol{y}} \cos f_1 \tag{5.31}$$

$$\hat{\boldsymbol{v}}_1 = \hat{\boldsymbol{h}}_1 \times \hat{\boldsymbol{n}}_i = -(\hat{\boldsymbol{x}} \cos q_1 \cos f_1 + \hat{\boldsymbol{y}} \cos q_1 \sin f_1 + \hat{\boldsymbol{z}} \sin q_1) \tag{5.32}$$

When the incident wave is horizontally polarised, $\hat{\boldsymbol{a}}$ in (5.14) and (5.15) equal to $\hat{\boldsymbol{h}}$. The scattered and depolarised fields are given respectively by

$$\begin{aligned}{}^1E^s_{hh1} &= \hat{\boldsymbol{h}}_{s1} \cdot {}^1\boldsymbol{E}^S_1 \\ &= {}^1M_1\left[R_{\|}\left(\hat{\boldsymbol{h}}_{s1}\cdot\hat{\boldsymbol{n}}_i\right)\left(\hat{\boldsymbol{h}}_{\boldsymbol{1}}\cdot\hat{\boldsymbol{n}}_{s1}\right)+R_{\perp}\left(\hat{\boldsymbol{v}}_{s1}\cdot\hat{\boldsymbol{n}}_i\right)\left(\hat{\boldsymbol{v}}_{\boldsymbol{1}}\cdot\hat{\boldsymbol{n}}_{s1}\right)\right] \\ &\equiv {}^1K_1{}^1I_1E_oU_{hh1}\end{aligned} \tag{5.33}$$

$$\begin{aligned}{}^1E^s_{hv1} &= \hat{\boldsymbol{v}}_{s1} \cdot {}^1\boldsymbol{E}^S_1 \\ &= {}^1M_1\left[R_{\|}\left(\hat{\boldsymbol{v}}_{s1}\cdot\hat{\boldsymbol{n}}_i\right)\left(\hat{\boldsymbol{h}}_{\boldsymbol{1}}\cdot\hat{\boldsymbol{n}}_{s1}\right)-R_{\perp}\left(\hat{\boldsymbol{h}}_{s1}\cdot\hat{\boldsymbol{n}}_i\right)\left(\hat{\boldsymbol{v}}_{\boldsymbol{1}}\cdot\hat{\boldsymbol{n}}_{s1}\right)\right] \\ &\equiv {}^1K_1{}^1I_1E_oU_{hv1}\end{aligned} \tag{5.34}$$

where ${}^1M_1 = {}^1K_1{}^1I_1E_oq_1|q_{z1}| \Big/ \left\{\left[\left(\hat{\boldsymbol{n}}_1\cdot\hat{\boldsymbol{h}}_{s1}\right)^2+\left(\hat{\boldsymbol{n}}_i\cdot\hat{\boldsymbol{v}}_{s1}\right)^2\right]k_1q_{z1}\right\}$. When the incident wave is vertically polarized, the scattered fields are obtained from (5.33) and (5.34) by an interchange of $\hat{\boldsymbol{v}}_{\boldsymbol{1}}$ with $\hat{\boldsymbol{h}}_{\boldsymbol{1}}$ and $\hat{\boldsymbol{v}}_{\boldsymbol{s1}}$ with $\hat{\boldsymbol{h}}_{s1}$ (detail Appendix G):

$$\begin{aligned}{}^1E^s_{vv1} &= \hat{\boldsymbol{h}}_{s1} \cdot {}^1\boldsymbol{E}^{\boldsymbol{S}}_{\boldsymbol{1}} \\ &= {}^1M_1\left[R_{\|}\left(\hat{\boldsymbol{v}}_{s1}\cdot\hat{\boldsymbol{n}}_i\right)\left(\hat{\boldsymbol{v}}_1\cdot\hat{\boldsymbol{n}}_{s1}\right)+R_{\perp}\left(\hat{\boldsymbol{h}}_{s1}\cdot\hat{\boldsymbol{n}}_i\right)\left(\hat{\boldsymbol{h}}_1\cdot\hat{\boldsymbol{n}}_{s1}\right)\right] \\ &\equiv {}^1K_1{}^1I_1E_oU_{vv1}\end{aligned} \tag{5.35}$$

$$\begin{aligned}{}^1E^s_{hv} &= \hat{\boldsymbol{v}}_{s1} \cdot {}^1\boldsymbol{E}^S_1 \\ &= M_1\left[R_{\|}\left(\hat{\boldsymbol{h}}_{s1}\cdot\hat{\boldsymbol{n}}_i\right)\left(\hat{\boldsymbol{v}}_1\cdot\hat{\boldsymbol{n}}_{s1}\right)-R_{\perp}\left(\hat{\boldsymbol{v}}_{s1}\cdot\hat{\boldsymbol{n}}_i\right)\left(\hat{\boldsymbol{h}}_1\cdot\hat{\boldsymbol{n}}_{s1}\right)\right] \\ &\equiv {}^1K_1{}^1I_1E_oU_{hv1}\end{aligned} \tag{5.36}$$

5.2.1.2 Scattering field on medium 2

In the same way, the scattered field in medium 2 (burnt coal seam) is obtained by substituting (5.16) and (5.17) to (5.1):

$${}^1\boldsymbol{E}^S_2 = {}^1K_2\hat{\boldsymbol{n}}^{\boldsymbol{t}}_{s1}\times\int\left[\hat{\boldsymbol{n}}_2\times\boldsymbol{E}-h_2\hat{\boldsymbol{n}}^{\boldsymbol{t}}_{\boldsymbol{s1}}\times\left(\hat{\boldsymbol{n}}_2\times\boldsymbol{H}\right)\right]\exp\left[j\left(k_{\boldsymbol{2}}\hat{\boldsymbol{n}}^{\boldsymbol{t}}_{\boldsymbol{s1}}-k_1\hat{\boldsymbol{n}}_i\right)\cdot\boldsymbol{r}'\right]d\boldsymbol{S}' \tag{5.37}$$

where ${}^1K_2 = -jk_2\exp\left(-jk_2R_1\right)/\left(4pR_1\right)$, $R_1 = x\sec q_{t1}$ and x is the thickness of burnt coal seam.

$$\hat{\boldsymbol{n}}_{s1}^{t} = \hat{\boldsymbol{x}}\sin q_{s1}^{t}\cos f_{s1}^{t} + \hat{\boldsymbol{y}}\sin q_{s1}^{t}\sin f_{s1}^{t} - \hat{\boldsymbol{z}}\cos q_{s1}^{t} \tag{5.38}$$

By applying the stationary-phase approximation in (5.37) with

$$Q = \left(k_2\hat{\boldsymbol{n}}_{s1}^{t} - k_1\hat{\boldsymbol{n}}_i\right)\cdot\hat{\boldsymbol{r}}' \equiv \bar{q}_{x1}x' + \bar{q}_{y1}y' + \bar{q}_{z1}z' \tag{5.39}$$

Thus we obtained

$$\bar{q}_{x1} = k_2\sin q_{s1}^{t}\cos f_{s1}^{t} - k_1\sin q_1\cos f_1 \tag{5.40}$$

$$\bar{q}_{y1} = k_2\sin q_{s1}^{t}\sin f_{s1}^{t} - k_1\sin q_1\sin f_1 \tag{5.41}$$

$$\bar{q}_{z1} = -k_2\cos q_{s1}^{t} + k_1\cos q_1 \tag{5.42}$$

Then the surface slopes can be replaced by

$$\frac{\partial z'}{\partial x'} = -\frac{\bar{q}_{x1}}{\bar{q}_{z1}} \tag{5.43}$$

$$\frac{\partial z'}{\partial y'} = -\frac{\bar{q}_{y1}}{\bar{q}_{z1}} \tag{5.44}$$

By this replacement, $\hat{\boldsymbol{n}}_2 \times \boldsymbol{H}$ and $\hat{\boldsymbol{n}}_2 \times \boldsymbol{E}$ become in dependent to the integration variables and (5.37) can be rewritten as

$${}^{1}\boldsymbol{E}_{2}^{s} = {}^{1}K_2\hat{\boldsymbol{n}}_{s1}^{t}\times\left(\hat{\boldsymbol{n}}_2\times\boldsymbol{E} - h_2\hat{\boldsymbol{n}}_{s1}^{t}\times\left(\hat{\boldsymbol{n}}_2\times\boldsymbol{H}\right)\right){}^{1}I_2 \tag{5.45}$$

where

$${}^{1}I_2 = \int \exp\left[j\left(k_2\hat{\boldsymbol{n}}_{s1}^{t} - k_1\hat{\boldsymbol{n}}_i\right)\cdot\boldsymbol{r}'\right]dS' \tag{5.46}$$

In the similar way to scattered fields in the medium 1, the scattered fields in medium 2 will be obtained as follows (detail Appendix H)

$$\begin{aligned} {}^{1}E_{hh2}^{s} &= -{}^{1}M_2\left(T_{\perp}\left(\hat{\boldsymbol{v}}_{s1}^{t}\cdot\hat{\boldsymbol{n}}_i\right)\left(\hat{\boldsymbol{v}}_1\cdot\hat{\boldsymbol{n}}_{s1}^{t}\right) + T_{\parallel}\left(\hat{\boldsymbol{h}}_{s1}^{t}\cdot\hat{\boldsymbol{n}}_i\right)\left(\hat{\boldsymbol{h}}_1\cdot\hat{\boldsymbol{n}}_{s1}^{t}\right)h_2/h_1\right) \\ &\equiv {}^{1}K_2\,{}^{1}I_2E_oD_{hh1} \end{aligned} \tag{5.47}$$

$$
\begin{aligned}
{}^1E^s_{vh2} &= {}^1M_2\left(T_\perp\left(\hat{\boldsymbol{h}}^{\boldsymbol{t}}_{\boldsymbol{s1}}\cdot\hat{\boldsymbol{n}}_i\right)\left(\hat{\boldsymbol{v}}_1\cdot\hat{\boldsymbol{n}}^{\boldsymbol{t}}_{\boldsymbol{s1}}\right)-T_\parallel\left(\hat{\boldsymbol{v}}^{\boldsymbol{t}}_{\boldsymbol{s1}}\cdot\hat{\boldsymbol{n}}_i\right)\left(\hat{\boldsymbol{h}}_{\boldsymbol{1}}\cdot\hat{\boldsymbol{n}}^{\boldsymbol{t}}_{\boldsymbol{s1}}\right)h_2/h_1\right)\\
&\equiv {}^1K_2\,{}^1I_2E_oD_{vh2}
\end{aligned}
\tag{5.48}
$$

$$
\begin{aligned}
{}^1E^s_{vv2} &= -{}^1M_2\left(T_\perp\left(\hat{\boldsymbol{h}}^{\boldsymbol{t}}_{\boldsymbol{s1}}\cdot\hat{\boldsymbol{n}}_i\right)\left(\hat{\boldsymbol{h}}_1\cdot\hat{\boldsymbol{n}}^{\boldsymbol{t}}_{\boldsymbol{s1}}\right)+T_\parallel\left(\hat{\boldsymbol{v}}^{\boldsymbol{t}}_{\boldsymbol{s1}}\cdot\hat{\boldsymbol{n}}_i\right)\left(\hat{\boldsymbol{v}}_{\boldsymbol{1}}\cdot\hat{\boldsymbol{n}}^{\boldsymbol{t}}_{\boldsymbol{s1}}\right)h_2/h_1\right)\\
&\equiv {}^1K_2\,{}^1I_2E_oD_{vv2}
\end{aligned}
\tag{5.49}
$$

$$
\begin{aligned}
{}^1E^s_{hv2} &= {}^1M_2\left(T_\perp\left(\hat{\boldsymbol{v}}^{\boldsymbol{t}}_{\boldsymbol{s1}}\cdot\hat{\boldsymbol{n}}_i\right)\left(\hat{\boldsymbol{h}}_{\boldsymbol{1}}\cdot\hat{\boldsymbol{n}}^{\boldsymbol{t}}_{\boldsymbol{s1}}\right)-T_\parallel\left(\hat{\boldsymbol{h}}^{\boldsymbol{t}}_{\boldsymbol{s1}}\cdot\hat{\boldsymbol{n}}_i\right)\left(\hat{\boldsymbol{v}}_{\boldsymbol{1}}\cdot\hat{\boldsymbol{n}}^{\boldsymbol{t}}_{\boldsymbol{s1}}\right)h_2/h_1\right)\\
&\equiv {}^1K_2\,{}^1I_2E_oD_{hv2}
\end{aligned}
\tag{5.50}
$$

where

$$\hat{\boldsymbol{h}}^{\boldsymbol{t}}_{\boldsymbol{s1}} = \hat{\boldsymbol{n}}^{\boldsymbol{t}}_{\boldsymbol{s1}}\times\hat{\boldsymbol{v}}^{\boldsymbol{t}}_{\boldsymbol{s1}} = -\hat{\boldsymbol{x}}\sin f^t_{s1}+\hat{\boldsymbol{y}}\cos f^t_{s1} \tag{5.51}$$

$$\hat{\boldsymbol{v}}^{\boldsymbol{t}}_{\boldsymbol{s1}} = -\left(\hat{\boldsymbol{x}}\cos q^t_{s1}\cos f^t_{s1}+\hat{\boldsymbol{y}}\cos q^t_{s1}\sin f^t_{s1}+\hat{\boldsymbol{z}}\sin q^t_{s1}\right) \tag{5.52}$$

$${}^1M_2 = 2\,{}^1K_2\,{}^1I_2\,E_o\left(k_2-k_1\left(\hat{\boldsymbol{n}}_i\cdot\hat{\boldsymbol{n}}^{\boldsymbol{t}}_{\boldsymbol{s1}}\right)\right)\Big/\left(qD_2\overline{D}_2\right) \tag{5.53}$$

$$\overline{q}_1^{\,2} = \overline{q}^{\,2}_{x1}+\overline{q}^{\,2}_{y1}+\overline{q}^{\,2}_{z1} \tag{5.54}$$

$$\overline{D}_2 = \overline{q}_{z1}D_2\Big/\left|\overline{q}_{z1}\right| \tag{5.55}$$

$$D_2 = \left|\hat{\boldsymbol{n}}_i\times\hat{\boldsymbol{n}}^{\boldsymbol{t}}_{\boldsymbol{s1}}\right| \tag{5.56}$$

5.2.2 Scattering field on peat surface or interface 2

To solving the problem of scattering from a perfectly conducting random surface, first, the determination of the surface current is done. To simplify the calculation, the surface current estimate is shown as (Gotoh *et al.* 1993)

$$\boldsymbol{J}(\boldsymbol{r}) = \hat{\boldsymbol{n}}_3\times\boldsymbol{H}^m \tag{5.57}$$

where $m = 2, 4, 6, \cdots$. $\hat{\boldsymbol{n}}_3$ is the unit normal vector to the surface of medium 3 or perfectly conducting medium (see figure 5.1), $\boldsymbol{H}^m$ is the incident magnetic field. The integration is over the illuminated area of the surface. To obtain an expression for the surface current, consider an incident horizontally and vertically polarized fields of the form

$$\begin{aligned} \boldsymbol{E}_h^m &= \hat{\boldsymbol{y}} E_{hp}^m \exp(-j\boldsymbol{k}\cdot\boldsymbol{r}) = \hat{\boldsymbol{y}} E_{hp}^m \exp(-jk_2 x \sin q_m + jk_2 z \cos q_m) \\ &= \hat{\boldsymbol{y}} E_{hp}^m \exp(-jk_{xm}x - jk_{zm}z) \end{aligned} \tag{5.58}$$

$$\boldsymbol{E}_v^m = E_{vp}^m (\hat{\boldsymbol{x}} \cos q_m + \hat{\boldsymbol{z}} \sin q_m) \exp(-jk_{xm}x - jk_{zm}z) \tag{5.59}$$

where p is h for horizontal and v for vertical, $E_{hp}^m = {}^{m-1}E_{hp2}^s$ and $E_{vp}^m = {}^{m-1}E_{vp2}^s$ are the field amplitudes at surface of medium 3 that are obtained from the scattered fields in medium 2, refer (5.47) to (5.50). q_m is the angle of incidence, k_2 is the wavenumber in medium 2 or burnt coal seam, and $j = \sqrt{-1}$. Next, let

$$\boldsymbol{n} = \hat{\boldsymbol{n}}_3 / \cos c = -\hat{\boldsymbol{x}} Z_x - \hat{\boldsymbol{y}} Z_y + \hat{\boldsymbol{z}} \tag{5.60}$$

$$\cos c = \left[1 + (Z_x)^2 + (Z_y)^2\right]^{-1/2} \tag{5.61}$$

where Z_x and Z_y are the partial derivatives of the surface of medium 3 that is shown by $z_2(x, y)$. Then (5.57) can be written as

$$\boldsymbol{J}_m = \frac{\boldsymbol{J}}{\cos c} = \frac{\hat{\boldsymbol{n}}_3}{\cos c} \times \boldsymbol{H}^m = \boldsymbol{n} \times \boldsymbol{H}^m \tag{5.62}$$

It is effectively the Kirchhoff approximation to the surface-current density (Gotoh *et al.* 1993). The expression for the surface current density for horizontal polarisation is

$$\boldsymbol{J}_h = \boldsymbol{J}_m = \frac{E_{hp}^m}{h_2} \begin{bmatrix} -Z_y \sin q_m \\ Z_x \sin q_m + \cos q_m \\ Z_y \cos q_m \end{bmatrix} \exp(-jk_{xm}x - jk_{zm}z) \tag{5.63}$$

and for vertical polarisation is

$$\boldsymbol{J}_v = \frac{E_{vp}^{m}}{\eta_2}\begin{bmatrix} 1 \\ 0 \\ Z_x \end{bmatrix}\exp\left(-jk_{xm}x - jk_{zm}z\right) \tag{5.64}$$

The derivation of (5.63) and (5.64) are shown in Appendix K. In accordance with the Stratton-Chu integral (Stratton 1941), the far-zone scattered fields from medium 3 (a perfectly conductor) is shown as

$$^{m}\boldsymbol{E}_{3}^{S}\left(\hat{\boldsymbol{n}}_{sm}\right) = -C_m\eta_2\hat{\boldsymbol{n}}_{sm} \times \int_{A_o} \hat{\boldsymbol{n}}_{sm} \times \boldsymbol{J}_p \exp\left(jk_2\hat{\boldsymbol{n}}_{sm}\cdot\boldsymbol{r}\right)dxdy \tag{5.65}$$

where $C_m = \left(jk_2/4\pi R_m\right)\exp\left(-jk_2R_m\right)$ and $R_m = \xi\sec\theta_m$. ξ is the thickness of burnt coal seam that is measured from burnt coal seam surface ($z=0$) to depth of a perfectly conducting medium ($z=\xi$). $\hat{\boldsymbol{n}}_{sm}$ is the unit vector pointing in the direction of observation, A_o is the illuminated area, η_2 is the intrinsic impedance of the burnt coal seam, $\boldsymbol{J}_p$ is either $\boldsymbol{J}_h$ or $\boldsymbol{J}_v$, $k_2\hat{\boldsymbol{n}}_{sm} = \hat{\boldsymbol{x}}k_2\sin\theta_m + \hat{\boldsymbol{z}}k_2\cos\theta_m = -\hat{\boldsymbol{x}}k_{xm} - \hat{\boldsymbol{z}}k_{zm}$ and k_2 is the wavenumber in medium 2. That can be rewritten as

$$\begin{aligned} ^{m}\boldsymbol{E}_{3}^{S}\left(\hat{\boldsymbol{n}}_{sm}\right) &= -C_m\eta_2\int_{A_o}\left[\hat{\boldsymbol{n}}_{sm}\left(\hat{\boldsymbol{n}}_{sm}\cdot\boldsymbol{J}_p\right) - \boldsymbol{J}_p\right]\exp\left(jk\hat{\boldsymbol{n}}_{sm}\cdot\boldsymbol{r}\right)dxdy \\ &= -C_m\eta_2\int_{A_o}\left[\hat{\boldsymbol{n}}_{sm}\left(\hat{\boldsymbol{n}}_{sm}\cdot\boldsymbol{J}_p\right) - \boldsymbol{J}_p\right]\exp\left(-jk_{xm}x - jk_{zm}z\right)dxdy \end{aligned} \tag{5.66}$$

For horizontal polarisation,

$$^{m}E_{hh}^{s} = \hat{\boldsymbol{y}}\cdot{}^{m}\boldsymbol{E}_{3}^{S}\left(\hat{\boldsymbol{n}}_{sm}\right) = C_mE_{hp}^{m}\int dxdy\left(Z_x\sin\theta_m + \cos\theta_m\right)\exp\left(-2jk_{xm}x - 2jk_{zm}z\right) \tag{5.67}$$

Integrating by parts in (5.67) and ignoring edge effects, we obtain

$$^{m}E_{hh}^{s} = C_mE_{hp}^{m}\sec\theta_m\int dxdy\exp\left(-2jk_{xm}x - 2jk_{zm}z\right) \tag{5.68}$$

Similarly, we can show that for the vertically polarised case the scattered field is

$$ {}^{m}E^{s}{}_{vv} = C_m E_{vp}^{m} \sec q_m \int dxdy \exp\left(-2jk_{xm}x - 2jk_{zm}z\right) \qquad (5.69) $$

Upon comparing (5.68) and (5.69), these equation are seen the same, it is indicating that for a perfectly conducting surface, there is no polarisation difference between vertically and horizontally polarised fields.

The cross-polarised scattered fields ${}^{m}E_{hv}^{S}$ and ${}^{m}E_{vh}^{S}$ are computed by taking $\hat{\boldsymbol{y}} \cdot {}^{m}\boldsymbol{E}_{3}^{S}(\hat{\boldsymbol{n}}_{sm})$ and $\sec q_m \hat{\boldsymbol{x}} \cdot {}^{m}\boldsymbol{E}_{3}^{S}(\hat{\boldsymbol{n}}_{sm})$ respectively, where respectively $\boldsymbol{J}_v$ and $\boldsymbol{J}_h$ are used in the field expression to obtain. Hence ${}^{m}E_{hv}^{s} = {}^{m}E_{vh}^{s} = 0$ is obtained. Now the polarised scattered fields were obtained and its will be used to calculated scattered fields at the boundary of medium 1 and 2 or interface 1 with distance R_m from the surface of perfectly conducting medium that will be discussed in the next section.

5.2.3 Scattering field on burnt coal seam surface (2) or interface 1

5.2.3.1 Scattering field on medium 2

The far zone scattered fields in medium 2 can be shown as

$$ {}^{m}\boldsymbol{E}_{2}^{S} = {}^{m}K_2 \hat{\boldsymbol{n}}_{sm} \times \int \left(\hat{\boldsymbol{n}}_2 \times \boldsymbol{E} - h_2 \hat{\boldsymbol{n}}_{sm} \times (\hat{\boldsymbol{n}}_2 \times \boldsymbol{H})\right) \exp\left(jk_2(\hat{\boldsymbol{n}}_{sm} - \hat{\boldsymbol{n}}_{im}) \cdot \boldsymbol{r}'\right) dS' \qquad (5.70) $$

where ${}^{m}K_2 = -jk_2 \exp(-jk_2R_m)/(4pR_m)$, $m = 3, 5, \cdots$ and

$$ \hat{\boldsymbol{n}}_{sm} = \hat{\boldsymbol{x}} \sin q_{sm} \cos f_{sm} + \hat{\boldsymbol{y}} \sin q_{sm} \sin f_{sm} - \hat{\boldsymbol{z}} \cos q_{sm} \qquad (5.71) $$

$$ \hat{\boldsymbol{n}}_{im} = \hat{\boldsymbol{x}} \sin q_m \cos f_m + \hat{\boldsymbol{y}} \sin q_m \sin f_m + \boldsymbol{z} \cos q_m \qquad (5.72) $$

Similarly with the previous derivation in section 5.2.1.1 and 5.2.1.2, we must simplify (5.70) using stationary-phase approximation, where phase Q of (5.70);

$$Q = k_2\left(\hat{n}_{sm} - \hat{n}_{im}\right)\cdot r' \equiv \bar{q}_{xm}x' + \bar{q}_{ym}y' + \bar{q}_{zm}z' \qquad (5.73)$$

where

$$\bar{q}_{xm} = k_2\left(\sin q_{sm}\cos f_{sm} - \sin q_m \cos f_m\right) \qquad (5.74)$$

$$\bar{q}_{ym} = k_2\left(\sin q_{sm}\sin f_{sm} - \sin q_m \sin f_m\right) \qquad (5.75)$$

$$\bar{q}_{zm} = -k_2\left(\cos q_{sm} + \cos q_m\right) \qquad (5.76)$$

by considering the stationary condition of the phase Q, the partial derivatives of the surface slopes can be replaced by the components of the phase as

$$\frac{\partial z'}{\partial x'} = -\frac{\bar{q}_{xm}}{\bar{q}_{zm}} \qquad (5.77)$$

$$\frac{\partial z'}{\partial y'} = -\frac{\bar{q}_{ym}}{\bar{q}_{zm}} \qquad (5.78)$$

Then the expression for ${}^{m}\boldsymbol{E}_2^{S}$ can be rewritten under the approximation as

$${}^{m}\boldsymbol{E}_2^{S} = K\hat{\boldsymbol{n}}_{sm}\times\left(\hat{\boldsymbol{n}}_2\times\boldsymbol{E} - h_2\hat{\boldsymbol{n}}_{sm}\times\left(\hat{\boldsymbol{n}}_2\times\boldsymbol{H}\right)\right)\,{}^{m}I_2 \qquad (5.79)$$

where

$${}^{m}I_2 = \int \exp\left(jk_2\left(\hat{\boldsymbol{n}}_{sm} - \hat{\boldsymbol{n}}_{im}\right)\cdot\boldsymbol{r}'\right)\,dS' \qquad (5.80)$$

Let we defined $\hat{\boldsymbol{v}}_m$, $\hat{\boldsymbol{h}}_m$ and $\hat{\boldsymbol{v}}_{sm}$, $\hat{\boldsymbol{h}}_{sm}$ be unit polarisation vectors for the incident and scattered vertical and horizontal wave respectively as

$$\hat{\boldsymbol{h}}_{sm} = \hat{f}_{\boldsymbol{sm}} = -\hat{\boldsymbol{x}}\sin f_{sm} + \hat{\boldsymbol{y}}\cos f_{sm} \qquad (5.81)$$

$$\hat{\boldsymbol{v}}_{sm} = \hat{q}_{\boldsymbol{sm}} = -\left(\hat{\boldsymbol{x}}\cos q_{sm}\cos f_{sm} + \hat{\boldsymbol{y}}\cos q_{sm}\sin f_{sm} + \hat{\boldsymbol{z}}\sin q_{sm}\right) \qquad (5.82)$$

$$\hat{\boldsymbol{h}}_m = f_{\boldsymbol{m}} = -\hat{\boldsymbol{x}}\sin f_m + \hat{\boldsymbol{y}}\cos f_m \qquad (5.83)$$

$$\hat{\boldsymbol{v}}_m = \hat{q}_{\boldsymbol{m}} = \hat{\boldsymbol{x}}\cos q_m \cos f_m + \hat{\boldsymbol{y}}\cos q_m \sin f_m - \hat{\boldsymbol{z}}\sin q_m \qquad (5.84)$$

The scattered polarised and depolarised fields are obtained

$$\begin{aligned} {}^m E^s_{hh2} &= {}^m M_2 \left(R_{\parallel} \left(\hat{\boldsymbol{h}}_{sm} \cdot \hat{\boldsymbol{n}}_{im} \right) \left(\hat{\boldsymbol{h}}_m \cdot \hat{\boldsymbol{n}}_{sm} \right) + R_{\perp} \left(\hat{\boldsymbol{v}}_{sm} \cdot \hat{\boldsymbol{n}}_{im} \right) \left(\hat{\boldsymbol{v}}_m \cdot \hat{\boldsymbol{n}}_{sm} \right) \right) \\ &\equiv {}^m K_2 {}^m I_2{}^{m-1} E^S_{hh} D_{hhm} \end{aligned} \qquad (5.85)$$

$$\begin{aligned} {}^m E^s_{vh2} &= {}^m M_2 \left(R_{\parallel} \left(\hat{\boldsymbol{v}}_{sm} \cdot \hat{\boldsymbol{n}}_{im} \right) \left(\hat{\boldsymbol{h}}_m \cdot \hat{\boldsymbol{n}}_{sm} \right) - R_{\perp} \left(\hat{\boldsymbol{h}}_{sm} \cdot \hat{\boldsymbol{n}}_{im} \right) \left(\hat{\boldsymbol{v}}_m \cdot \hat{\boldsymbol{n}}_{sm} \right) \right) \\ &\equiv {}^m K_2 {}^m I_2{}^{m-1} E^S_{vh} D_{vhm} = 0 \end{aligned} \qquad (5.86)$$

Where ${}^m M_2 = {}^m K_2 \; {}^m I_2 \; {}^{m-1} E^s_{pq} \overline{q}_m \left| \overline{q}_m \right| \Big/ \left(\left(\left(\hat{\boldsymbol{n}}_2 \cdot \hat{\boldsymbol{h}}_{sm} \right)^2 + \left(\hat{\boldsymbol{n}}_{im} \cdot \hat{\boldsymbol{v}}_{sm} \right)^2 \right) k_2 \overline{q}_{zm} \right)$. When the incident wave is vertically polarised, the scattered fields can be obtained from (5.85) and (5.86) by an interchange of $\hat{\boldsymbol{v}}_m$ with $\hat{\boldsymbol{h}}_m$ and $\hat{\boldsymbol{v}}_{sm}$ with $\hat{\boldsymbol{h}}_{sm}$;

$$\begin{aligned} {}^m E^s_{vv2} &= {}^m M_2 \left(R_{\parallel} \left(\hat{\boldsymbol{v}}_{sm} \cdot \hat{\boldsymbol{n}}_{im} \right) \left(\hat{\boldsymbol{v}}_m \cdot \hat{\boldsymbol{n}}_{sm} \right) + R_{\perp} \left(\hat{\boldsymbol{h}}_{sm} \cdot \hat{\boldsymbol{n}}_{im} \right) \left(\hat{\boldsymbol{h}}_m \cdot \hat{\boldsymbol{n}}_{sm} \right) \right) \\ &\equiv {}^m K_2 {}^m I_2{}^{m-1} E^S_{vv} D_{vvm} \end{aligned} \qquad (5.87)$$

$$\begin{aligned} {}^m E^s_{hv2} &= {}^m M_2 \left(R_{\parallel} \left(\hat{\boldsymbol{h}}_{sm} \cdot \hat{\boldsymbol{n}}_{im} \right) \left(\hat{\boldsymbol{v}}_m \cdot \hat{\boldsymbol{n}}_{sm} \right) - R_{\perp} \left(\hat{\boldsymbol{v}}_{sm} \cdot \hat{\boldsymbol{n}}_{im} \right) \left(\hat{\boldsymbol{h}}_m \cdot \hat{\boldsymbol{n}}_{sm} \right) \right) \\ &\equiv {}^m K_2 {}^m I_2{}^{m-1} E^S_{hv} D_{hvm} = 0 \end{aligned} \qquad (5.88)$$

The derivation of these equations is shown in Appendix I.

5.2.3.2 Scattering field on medium 1

In the same way with the section above, the far zone scattered fields in medium 1 (air) is obtained

$${}^m \boldsymbol{E}^{\boldsymbol{S}}_{\boldsymbol{1}} = {}^m K_1 \hat{\boldsymbol{n}}^{\boldsymbol{t}}_{\boldsymbol{sm}} \times \int \left(\hat{\boldsymbol{n}}_1 \times \boldsymbol{E} - h_1 \hat{\boldsymbol{n}}^{\boldsymbol{t}}_{\boldsymbol{sm}} \times \left(\hat{\boldsymbol{n}}_1 \times \boldsymbol{H} \right) \right) \exp\left(j \left(k_1 \hat{\boldsymbol{n}}^{\boldsymbol{t}}_{\boldsymbol{sm}} - k_2 \hat{\boldsymbol{n}}_{im} \right) \cdot \boldsymbol{r}' \right) dS' \qquad (5.89)$$

where $m = 3, 5, \cdots$ and

$$\boldsymbol{n}^t_{sm} = \hat{\boldsymbol{x}} \sin q^t_{sm} \cos f^t_{sm} + \hat{\boldsymbol{y}} \sin q^t_{sm} \sin f^t_{sm} + \hat{\boldsymbol{z}} \cos q^t_{sm} \qquad (5.90)$$

By applying the stationary-phase approximation in (5.89) with

$$Q = \left(k_1 \hat{\boldsymbol{n}}_{sm}^{t} - k_2 \hat{\boldsymbol{n}}_{im}\right)\cdot \boldsymbol{r}' = q_{xm} x' + q_{ym} y' + q_{zm} z' \tag{5.91}$$

where

$$q_{xm} = k_1 \sin\boldsymbol{q}_{sm}^{t} \cos f_{sm}^{t} - k_2 \sin\boldsymbol{q}_{m} \cos f_{m} \tag{5.92}$$

$$q_{ym} = k_1 \sin\boldsymbol{q}_{sm}^{t} \sin f_{sm}^{t} - k_2 \sin\boldsymbol{q}_{m} \sin f_{m} \tag{5.93}$$

$$q_{zm} = k_1 \cos\boldsymbol{q}_{sm}^{t} - k_2 \cos\boldsymbol{q}_{m} \tag{5.94}$$

Then the surface slopes can be replaced by

$$\frac{\partial z'}{\partial x'} = -\frac{q_{xm}}{q_{zm}} \tag{5.95}$$

$$\frac{\partial z'}{\partial y'} = -\frac{q_{ym}}{q_{zm}} \tag{5.96}$$

Then the expression for ${}^{m}\boldsymbol{E}_{1}^{S}$ can be rewritten under the approximation as

$${}^{m}\boldsymbol{E}_{1}^{S} = {}^{m}K_1 \hat{\boldsymbol{n}}_{sm}^{t} \times \left(\hat{\boldsymbol{n}}_1 \times \boldsymbol{E} - h_1 \hat{\boldsymbol{n}}_{sm}^{t} \times \left(\hat{\boldsymbol{n}}_1 \times \boldsymbol{H}\right)\right) {}^{m}I_1 \tag{5.97}$$

where

$${}^{m}I_1 = \int \exp\left(j\left(k_1 \hat{\boldsymbol{n}}_{sm}^{t} - k_2 \hat{\boldsymbol{n}}_{im}\right)\cdot \boldsymbol{r}'\right) dS' \tag{5.98}$$

Let we defined $\hat{\boldsymbol{v}}_m$, $\hat{\boldsymbol{h}}_m$ and $\hat{\boldsymbol{v}}_{sm}^{t}$, $\hat{\boldsymbol{h}}_{sm}^{t}$ be unit polarisation vectors for the incident and scattered vertical and horizontal wave respectively as (detail see Appendix J);

$$\begin{aligned} {}^{m}E_{hh1}^{s} &= -{}^{m}M_1\left(T_\perp\left(\hat{\boldsymbol{v}}_{sm}^{t}\cdot\hat{\boldsymbol{n}}_{im}\right)\left(\hat{\boldsymbol{v}}_{m}\cdot\hat{\boldsymbol{n}}_{sm}^{t}\right) + T_\parallel\left(\hat{\boldsymbol{h}}_{sm}^{t}\cdot\hat{\boldsymbol{n}}_{im}\right)\left(\hat{\boldsymbol{h}}_{m}\cdot\hat{\boldsymbol{n}}_{sm}^{t}\right)h_1/h_2\right) \\ &\equiv {}^{m}K_1 {}^{m}I_1^{m-1}E_{hh}^{S}U_{hhm} \end{aligned} \tag{5.99}$$

$$\begin{aligned} {}^{m}E_{vh1}^{s} &= {}^{m}M_1\left(T_\perp\left(\hat{\boldsymbol{h}}_{sm}^{t}\cdot\hat{\boldsymbol{n}}_{im}\right)\left(\hat{\boldsymbol{v}}_{m}\cdot\hat{\boldsymbol{n}}_{sm}^{t}\right) - T_\parallel\left(\hat{\boldsymbol{v}}_{sm}^{t}\cdot\hat{\boldsymbol{n}}_{im}\right)\left(\hat{\boldsymbol{h}}_{m}\cdot\hat{\boldsymbol{n}}_{sm}^{t}\right)h_1/h_2\right) \\ &\equiv {}^{m}K_1 {}^{m}I_1^{m-1}E_{vh}^{S}U_{vhm} = 0 \end{aligned} \tag{5.100}$$

$$\begin{aligned} {}^{m}E_{vv1}^{s} &= -{}^{m}M_1\left(T_\perp\left(\hat{\boldsymbol{h}}_{sm}^{t}\cdot\hat{\boldsymbol{n}}_{im}\right)\left(\hat{\boldsymbol{h}}_{m}\cdot\hat{\boldsymbol{n}}_{sm}^{t}\right) + T_\parallel\left(\hat{\boldsymbol{v}}_{sm}^{t}\cdot\hat{\boldsymbol{n}}_{im}\right)\left(\hat{\boldsymbol{v}}_{m}\cdot\hat{\boldsymbol{n}}_{sm}^{t}\right)h_1/h_2\right) \\ &\equiv {}^{m}K_1 {}^{m}I_1^{m-1}E_{vv}^{S}U_{vvm} \end{aligned} \tag{5.101}$$

$$
\begin{aligned}
{}^{m}E^{s}_{hv1} &= {}^{m}M_1\left(T_\perp\left(\hat{\boldsymbol{v}}^t_{sm}\cdot\hat{\boldsymbol{n}}_{im}\right)\left(\hat{\boldsymbol{h}}_m\cdot\hat{\boldsymbol{n}}^t_{sm}\right)-T_\parallel\left(\hat{\boldsymbol{h}}^t_{sm}\cdot\hat{\boldsymbol{n}}_{im}\right)\left(\hat{\boldsymbol{v}}_m\cdot\hat{\boldsymbol{n}}^t_{sm}\right)h_1/h_2\right) \\
&\equiv {}^{m}K_1\,{}^{m}I_1^{m-1}E^S_{hv}U_{hvm} = 0
\end{aligned}
\tag{5.102}
$$

where

$$
\hat{\boldsymbol{v}}^t_{sm} = \hat{q}^t_{sm} = \hat{\boldsymbol{x}}\cos q^t_{sm}\cos f^t_{sm} + \hat{\boldsymbol{y}}\cos q^t_{sm}\sin f^t_{sm} - \hat{\boldsymbol{z}}\sin q^t_{sm} \tag{5.103}
$$

$$
\hat{\boldsymbol{h}}^t_{sm} = \hat{f}^t_{sm} = -\hat{\boldsymbol{x}}\sin f^t_{sm} + \hat{\boldsymbol{y}}\cos f^t_{sm} \tag{5.104}
$$

$$
{}^{m}M_1 = {}^{m}K_1\,{}^{m}I_1^{m-1}E^s_{pq}\left(k_1 - k_2\left(\hat{\boldsymbol{n}}_{im}\cdot\hat{\boldsymbol{n}}^t_{sm}\right)\right)\Big/\left(q_m D_m \overline{D}_m\right) \tag{5.105}
$$

$$
q_m^2 = q_{xm}^2 + q_{ym}^2 + q_{zm}^2 \tag{5.106}
$$

$$
\overline{D}_m = q_z D_m \big/ \left|q_z\right| \tag{5.107}
$$

$$
D_m = \left|\hat{\boldsymbol{n}}_{im}\times\hat{\boldsymbol{n}}^t_{sm}\right| \tag{5.108}
$$

Now that all the field expressions are available, we are ready to compute the average scattered power and scattering coefficient.

5.2.4 Scattering coefficient

By derivation of scattered wave in medium 1 and 2, see figure 5.1, the total scattered field in medium 1 (air) is defined by summation of them and is shown as

$$
E^s_{pq} = {}^{1}E^s_{pq1} + {}^{3}E^s_{pq1} + {}^{5}E^s_{pq1} + \cdots = \sum_{n=0}^{\infty} {}^{2n+1}E^s_{pq1} \tag{5.109}
$$

Hence, mathematically the scattering coefficient in medium 1 (air) can be written as

$$
\begin{aligned}
s^o_{pq} &= \frac{4pR^2}{A_o}\frac{\mathrm{Re}\left(\left\langle\left|E^s_{pq}\right|^2\right\rangle\right)}{\mathrm{Re}\left(\left|E_o\right|^2\right)} = \frac{4pR^2}{A_o}\frac{\mathrm{Re}\left(\left\langle\left|\sum_{n=0}^{\infty} {}^{2m+1}E^s_{pq1}\right|^2\right\rangle\right)}{\mathrm{Re}\left(\left|E_o\right|^2\right)} \\
&\approx \frac{4pR^2}{A_o}\left\{\mathrm{Re}\left(\left\langle\left|{}^{1}E^S_{pq1}\right|^2\right\rangle\right)\Big/\mathrm{Re}\left(\left|E_o\right|^2\right) + \mathrm{Re}\left(\left\langle\left|{}^{3}E^S_{pq1}\right|^2\right\rangle\right)\Big/\mathrm{Re}\left(\left|E_o\right|^2\right)\right.
\end{aligned}
$$

$$+\operatorname{Re}\left(\left\langle\left|{}^{5}E_{pq1}^{S}\right|^{2}\right\rangle\right)\Big/\operatorname{Re}\left(\left|E_{o}\right|^{2}\right)+\cdots\Big\}$$
$$={}^{1}\sigma_{pq}^{o}+{}^{3}\sigma_{pq}^{o}+{}^{5}\sigma_{pq}^{o}+\cdots \tag{5.110}$$

where A_o is the illuminated area, R is the distance from the point of observation to the center of A_o, $\operatorname{Re}\{\cdots\}$ is the real part operator, and $\langle\cdots\rangle$ is the symbol for ensemble average.

To compute σ_{pq}^{o} for different polarization states, it is necessary to calculate the ensemble average of $\left|{}^{1}I_{m}\right|^{2}$. For example, the first term in (5.110), since the two integrals are similar, it is sufficient to show the computation of $\left\langle\left|{}^{1}I_{1}\right|^{2}\right\rangle$ for a Gaussian-distributed random surface with surface-height distribution of medium 2 or burnt coal seam:

$$p_{1}(z)=\left(2\pi\sigma_{1}^{2}\right)^{-\frac{1}{2}}\exp\left(-z^{2}/2\sigma_{1}^{2}\right) \tag{5.111}$$

where σ_{1}^{2} is the variance of surface heights of medium 2 or burnt coal seam. We have

$$\left\langle\left|{}^{1}I_{1}\right|^{2}\right\rangle=\iint\left\langle\exp\left[jk_{1}\left(\hat{\boldsymbol{n}}_{s1}-\hat{\boldsymbol{n}}_{i1}\right)\cdot\left(\boldsymbol{r}'-\boldsymbol{r}''\right)\right]\right\rangle dS'S'' \tag{5.112}$$

to express (5.112) in the rectangular coordinates shown in figure 5.1, note that

$$dS'=dx'dy'\big/\left(\hat{\boldsymbol{n}}_{1}\cdot\hat{\boldsymbol{z}}\right)=q_{1}dx'dy'\big/\left|q_{z1}\right| \tag{5.113}$$

Hence

$$\left\langle\left|{}^{1}I_{1}\right|^{2}\right\rangle=\frac{q_{1}^{2}}{q_{z1}^{2}}\iint\iint\exp\left[jq_{x1}\left(x'-x''\right)+jq_{y1}\left(y'-y''\right)\right]$$
$$\times\left\langle\exp\left[jq_{z1}\left(z\left(x',y'\right)-z\left(x'',y''\right)\right)\right]\right\rangle dx'dy'dx''dy'' \tag{5.114}$$

The factor $\langle\cdots\rangle$ in (5.114) is recognized as the joint characteristic function of $z(x',y')$ and $z(x'',y'')$. By assuming $z(x,y)$ to be a stationary Gaussian random process with zero mean, variance σ_{1}^{2}, and correlation coefficient ρ_{1}, the characteristic function is given by (Wu *et al.*

1988).

$$\langle \cdots \rangle = \exp\left(-q_{z1}^2 s_1^2 (1 - r_1)\right) \tag{5.115}$$

The correlation coefficient of a random process is a function of spatial variables. For stationary process, it depends only on the difference variables, $u = x' - x''$, $v = y' - y''$. Assume that the size of the illuminated area is $2\text{L} \times 2\text{L}$. In terms of the difference variables, (5.114) becomes

$$\left\langle \left| {}^1I_1 \right|^2 \right\rangle = \frac{q_1^2}{q_{z1}^2} \int_{-L}^{L} \int_{-L}^{L} \int_{-L-x''}^{L-x''} \int_{-L-y''}^{L-y''} \exp\left(jq_{x1}u + jq_{y1}v - q_{z1}^2 s_1^2 (1 - r_1)\right) du dv dx'' dy'' \tag{5.116}$$

This equation can be simplified by interchanging the order of integration, which leads to the following identity:

$$\int_{-L}^{L} \int_{-L-x''}^{L-x''} \exp(jq_{x1}u) f(u) du dx'' = \int_{-2L}^{2L} (2L - |u|) f(u) \exp(jq_{x1}u) du \tag{5.117}$$

The use of the above identity permits (5.116) to be written as

$$\left\langle \left| {}^1I_1 \right|^2 \right\rangle = \frac{q_1^2}{q_{z1}^2} \int_{-2L}^{2L} \int_{-2L}^{2L} (2L - |u|)(2L - |v|) \exp\left(jq_{x1}u + jq_{y1}v\right) \exp\left(-q_{z1}^2 s_1^2 (1 - r_1)\right) du dv \tag{5.118}$$

Further simplification requires additional assumptions. Two commonly used assumptions are that (a) the surface roughness is isotropic (b) $(q_{z1} r_1)^2$ is large so that the contribution to the integrals in (5.118) is significant only for small values of u and v. Then r_1 can be approximated by the first two terms of its Taylor series expansion about the origin. In this case, it is advantageous to change u, v to polar coordinates r and f. Upon ignoring $|u|$, $|v|$ in comparison with $2L$ and integrating f, (5.118) reduces to

$$\left\langle \left| {}^{1}I_{1} \right|^{2} \right\rangle = \frac{2pq_{1}^{2}}{q_{z1}^{2}} (2L)^{2} \int_{0}^{2L} J_{o}\left(r\left[q_{x1}^{2} + q_{y1}^{2}\right]^{\frac{1}{2}} \right) \exp\left[- q_{z1}^{2} s_{1}^{2} \left| r_{1}''(0) \right| r^{2}/2\right] rdr \tag{5.119}$$

where $J_o(\cdots)$ is the zeroth-order Bessel function, $r_1''(0)$ is the second derivative of r_1 evaluated at the origin, and $(2L)^2$ is the illuminated area A_o. Note that in (5.119), $s_1^2 \left| r_1''(0) \right|$ corresponds to the mean squared slope of the surface. Since the integrand of (5.119) is negligible for large values of r, no significant error results is we extend the upper limit to infinity. With this change in limit, the integrated result of (5.119) is

$$\left\langle \left| {}^{1}I_{1} \right|^{2} \right\rangle = \frac{2pA_{o}q_{1}^{2}}{q_{z1}^{4} s_{1}^{2} \left| r_{1}''(0) \right|} \exp\left(- \frac{q_{x1}^{2} + q_{y1}^{2}}{2q_{z1}^{2} s_{1}^{2} \left| r_{1}''(0) \right|} \right) \tag{5.120}$$

Upon substituting (5.120) into the product in the scattered-field expression for medium 1, we obtain

$$\left\langle {}^{1}E_{pq1}^{S} \cdot {}^{1}E_{pq1}^{S\,*} \right\rangle = \left| {}^{1}K_{1}E_{o}U_{pq1} \right|^{2} \left\langle \left| {}^{1}I_{1} \right|^{2} \right\rangle = \frac{\left| k_{1}E_{o}U_{pq1} \right|^{2}}{(4pR)^{2}} \left\langle \left| {}^{1}I_{1} \right|^{2} \right\rangle \tag{5.121}$$

Substituting (5.121) in (5.110), we obtain the reflected bistatic-scattering coefficient in medium 1 as

$${}^{1}s_{pq}^{o} = \frac{\left(k_{1}q_{1}\left|U_{pq1}\right|\right)^{2}}{2q_{z1}^{4} s_{1}^{2} \left| r_{1}''(0) \right|} \exp\left[- \frac{q_{x1}^{2} + q_{y1}^{2}}{2q_{z1}^{2} s_{1}^{2} \left| r_{1}''(0) \right|} \right] \tag{5.122}$$

Similarly, by considering the scattered field from medium 3 (peat as a perfectly conductor) or interface 2, the scattering coefficient in $m = 3,\ 5,\ 7,\ \cdots$ is obtained as

$$\begin{aligned} \left\langle {}^{m}E_{pqm}^{S} \cdot {}^{m}E_{pqm}^{S\,*} \right\rangle = & \left| {}^{m}K_{1}^{\,m-2} K_{2} E_{o} U_{pqm} D_{pqm-2} C_{m-1} \sec q_{m-1} \right|^{2} \left\langle \left| {}^{m}I_{1} \right|^{2} \right\rangle \left\langle \left| {}^{m-2}I_{2} \right|^{2} \right\rangle \\ & \cdot \left(\int dxdy \exp\left(-2jk_{2}\hat{\boldsymbol{n}}_{sm-1} \cdot \boldsymbol{r}'\right) \right)^{2} \end{aligned}$$

$$
\begin{aligned}
&= \frac{\left|k_1 E_o U_{pq1}\right|^2}{\left(4pR_m\right)^2}\left\langle\left|{}^m I_1\right|^2\right\rangle \cdot \frac{\left|k_2 D_{pqm-2}\right|^2}{\left(4pR_{m-2}\right)^2}\left\langle\left|{}^{m-2} I_2\right|^2\right\rangle \cdot \frac{\left|k_2 \sec q_{m-1}\right|^2}{\left(4pR_{m-1}\right)^2}\left\langle\left|{}^{m-1} I_2\right|^2\right\rangle \\
&= {}^m\ f_{pq}^1 \cdot {}^m\ f_{pq}^2 \cdot {}^m\ f_{pq}^3
\end{aligned}
\qquad (5.123)
$$

where

$$
\left\langle\left|{}^{m-1} I_2\right|^2\right\rangle = \int dxdy \exp\left(-4jk_2 \hat{\boldsymbol{n}}_{sm-1} \cdot \boldsymbol{r}'\right) \qquad (5.124)
$$

By referring to (5.122), ${}^m f_{pq}^1$ and ${}^m f_{pq}^2$ can be derived using the same manner. Then to calculate the average scattered power of the factor ${}^m f_{pq}^3$, the major interest here is in the incoherently scattered power, which what is usually measured. This means that we should subtract the mean or the coherent field from the express on given by (5.68) and (5.69) before we form the power expression. However, this is equivalent to computing the factor ${}^m f_{pq}^3$ in (5.123) and then subtracting an appropriate mean product or mean squared quantity from each other. In what follows, we shall proceed by considering only final factor in (5.123). For the purpose of illustration, we shall assume a Gaussian height distribution for the surface of medium 3 (peat) under consideration. The mean squared of (5.68) and (5.69) are

$$
\begin{aligned}
\left\langle {}^m E^S_{\substack{vv\\hh}}\, {}^m E^{S\,*}_{\substack{vv\\hh}} \right\rangle &= \left|C_m\right|^2 \left(E^m_{\substack{vv\\hh}}\right)^2 \sec^2 q_{m-1} \int dxdyd\bar{x}d\bar{y} \exp\left[2jk_x\left(\bar{x}-x\right)\right]\left\langle \exp\left[2jk_z\left(\bar{z}-z\right)\right]\right\rangle \\
&= \left|C_m\right|^2 \left(E^m_{\substack{vv\\hh}}\right)^2 \sec^2 q_{m-1} \int dxdyd x dV \exp\left[2jk_x x\right] \exp\left\{-4k_z^2 s_2^2\left[1-r_2\left(x, V\right)\right]\right\}
\end{aligned}
\qquad (5.125)
$$

where $x = x - \bar{x}$, $V = y - \bar{y}$, $r_2\left(x, V\right)$ is the surface-height autocorrelation function and s_2^2 is the variance of the surface of medium 3. To obtain the incoherent power, we subtract the mean-squared value $\left\langle {}^m E^S_{\substack{vv\\hh}} \right\rangle^2$ from it, yielding

$$
\begin{aligned}
\left\langle {}^m E^S_{\substack{vv\\hh}}\, {}^m E^{S\,*}_{\substack{vv\\hh}} \right\rangle - \left\langle {}^m E^S_{\substack{vv\\hh}} \right\rangle^2 &= \left|C_m\right|^2 \left(E^m_{\substack{vv\\hh}}\right)^2 \sec^2 q_{m-1} A_o \exp\left(-4k_z^2 s_2^2\right) \int dxdV \\
&\quad \cdot \exp\left(2jk_x x\right)\left[\exp\left(4k_z^2 s_2^2 r_2 - 1\right)\right]
\end{aligned}
\qquad (5.126)
$$

where A_o is the illuminated area. Hence the backscattering coefficient for the factor ${}^{m}f_{pq}^{3}$ of (5.123) can be obtained multiplying the power expression (5.126) by the factor $4\pi R \big/ A_o \operatorname{Re}\left(\left| {}^{1}E_{pq2}^{S} \right|^2 \right)$. Denoting the factor of backscattering coefficient by ${}^{m}f_{hh}^{3}$ and ${}^{m}f_{vv}^{3}$ for horizontal and vertical cases respectively, we have

$$
{}^{m}f_{\substack{vv\\hh}}^{3} \approx \frac{2k_2^2}{R_{m-1}^2 \cos^2\theta_{m-1}} \left\{ \exp\left(-4k_2^2\sigma_2^2\cos^2\theta_{m-1}\right) \sum_{n=1}^{\infty} \left[4k_2^2\sigma_2^2\cos^2\theta_{m-1}\right]^n \big/ n! \right\} \cdot W^{(n)}\left(2k_2\sin\theta_{m-1},\, 0\right) \tag{5.127}
$$

or

$$
{}^{m}f_{\substack{vv\\hh}}^{3} \approx \frac{8k_2^2 \exp\left(-2k_2^2\sigma_2^2\cos^2\theta_{m-1}\right)}{R_{m-1}^2\cos^2\theta_{m-1}} \sum_{n=1}^{\infty} \frac{W^{(n)}\left(2k_2\sin\theta_{m-1},\, 0\right)}{n!} \left[k_2^2\sigma_2^2\cos^2\theta_{m-1}\right]^n \cdot 2^{2n-2}\exp\left(-2k_2^2\sigma_2^2\cos^2\theta_{m-1}\right) \tag{5.128}
$$

where $W^{(n)}(U, V)$ is the roughness spectrum of the surface related to the nth power of the surface correlation function by Fourier transform as follows:

$$
W^{(n)}(U, V) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\xi dV \exp\left[-jU\xi - jVV\right] \rho^n(\xi, V) \qquad (n = 1, 2, \cdots) \tag{5.129}
$$

As an example for a Gaussian correlation function $\rho(\xi, V) = \exp\left[-\left(\xi^2 + V^2\right)/l\right]$, where l is pulse width, (5.127) takes the form

$$
{}^{m}f_{\substack{vv\\hh}}^{3} = \left(\frac{2k_2 l}{R_{m-1}\cos\theta_{m-1}} \right)^2 \exp\left[-\left(2k_2\sigma_2\cos\theta_{m-1}\right)^2\right] \cdot \sum_{n=1}^{\infty} 4^{m-1} \frac{\left(k_2\sigma_2\cos\theta_{m-1}\right)^{2n}}{n!\, n} \exp\left[\frac{-\left(kl\sin\theta_{m-1}\right)^2}{n}\right] \tag{5.130}
$$

To simplify the computation, in the derivation of ${}^{m}\sigma_{pq}^{o}$, the effects of shadowing and multiple scattering have been ignored. Hence the backscattering coefficient for

$m = 3,\ 5,\ 7,\ \cdots$ is

$$
\begin{aligned}
{}^{m}\sigma^{o}_{pq} \approx {} & \frac{\left(k_1\theta_m \left|U_{pqm}\right|\right)^2}{2\theta_{zm}^4 \sigma_1^2 \left|\rho_1''(0)\right|} \exp\left[-\frac{\theta_{xm}^2+\theta_{ym}^2}{2\theta_{zm}^2\sigma_1^2\left|\rho_1''(0)\right|}\right] \\
& \cdot \frac{\left(k_2\bar{\theta}_{m-2}\left|D_{pqm-2}\right|\right)^2}{2\bar{\theta}_{zm-2}^4\sigma_1^2\left|\rho_1''(0)\right|\left(R_{m-2}\right)^2} \exp\left[-\frac{\bar{\theta}_{xm-2}^2+\bar{\theta}_{ym-2}^2}{2\bar{\theta}_{zm-2}^2\sigma_1^2\left|\rho_1''(0)\right|}\right] \\
& \cdot \left(\frac{2k_2 l}{R_{m-1}\cos\theta_{m-1}}\right)^2 \exp\left[-\left(2k_2\sigma_2\cos\theta_{m-1}\right)^2\right] \\
& \cdot \sum_{n=1}^{\infty} 4^{n-1}\frac{\left(k_2\sigma_2\cos\theta_{m-1}\right)^{2n}}{n!\,n}\exp\left[\frac{-\left(kl\sin\theta_{m-1}\right)^2}{n}\right]
\end{aligned}
\tag{5.131}
$$

where $R_{m-1} = x\sec\theta_{m-1}$. x is the thickness of burnt coal seam. Finally, (5.122) and (5.131) are substituted to (5.110) and the total backscattering coefficient is obtained as

$$
\begin{aligned}
\sigma^{o}_{pq} = {} & {}^{1}\sigma^{o}_{pq} + {}^{3}\sigma^{o}_{pq} + {}^{5}\sigma^{o}_{pq} + \cdots \\
= {} & \frac{\left(k_1\theta_1\left|U_{pq1}\right|\right)^2}{2\theta_{z1}^4\sigma_1^2\left|\rho_1''(0)\right|}\exp\left[-\frac{\theta_{x1}^2+\theta_{y1}^2}{2\theta_{z1}^2\sigma_1^2\left|\rho_1''(0)\right|}\right] \\
& + \sum_{m=1}^{\infty}\left\{\frac{\left(k_1\theta_{2m+1}\left|U_{pq2m+1}\right|\right)^2}{2\theta_{z2m+1}^4\sigma_1^2\left|\rho_1''(0)\right|}\exp\left[-\frac{\theta_{x2m+1}^2+\theta_{y2m+1}^2}{2\theta_{z2m+1}^2\sigma_1^2\left|\rho_1''(0)\right|}\right]\right. \\
& \cdot \frac{\left(k_2\bar{\theta}_{2m-1}\left|D_{pq2m-1}\right|\right)^2}{2\bar{\theta}_{z2m-1}^4\sigma_1^2\left|\rho_1''(0)\right|\left(R_{2m-1}\right)^2}\exp\left[-\frac{\bar{\theta}_{x2m-1}^2+\bar{\theta}_{y2m-1}^2}{2\bar{\theta}_{z2m-1}^2\sigma_1^2\left|\rho_1''(0)\right|}\right] \\
& \cdot \left(\frac{2k_2 l}{R_{2m}\cos\theta_{2m}}\right)^2\exp\left[-\left(2k_2\sigma_2\cos\theta_{2m}\right)^2\right] \\
& \left.\cdot \sum_{n=1}^{\infty}4^{n-1}\frac{\left(k_2\sigma_2\cos\theta_{2m}\right)^{2n}}{n!\,n}\exp\left[\frac{-\left(kl\sin\theta_{2m}\right)^2}{n}\right]\right\}
\end{aligned}
\tag{5.132}
$$

Now the relationship between backscattering coefficient and the thickness of burnt coal seam is obtained. The computation results and its confirmation are discussed in the next section.

5.3 Results and Discussion

To obtain correlation between the like-polarised backscattering coefficient s^o and the thickness of burnt coal seam ξ, parameters of the burnt coal seam are specific permeability $m_r = 1$, dielectric constant of burnt coal seam in frequency of 1.275 GHz (JERS-1 SAR) is $e_1 = 2.5 - j0.1$ (see figure 1.7), wavelength $l = 23.5$ cm, incident angle $q_i = 38.7^o$, and ξ varied from 0 to 1m. Standard deviation s_1 of medium 2 (burnt coal seam) and medium 3 (peat) s_2 surfaces are assumed 0.3m (Nuraini 1999). By substituting these variables in (5.132), the backscattering coefficient s^o with respect to each the thickness of burnt coal seam ξ is obtained. This result is illustrated in figure 5.2 (hh-polarisation and vv-polarisation). This figure shows that increment in the thickness ξ of burnt coal seam was directly proportional to reduction in backscattering coefficient s^o. It implies that the burnt coal seam absorbed wave energy. In the same figure, the previous result is also shown (previous method). This result shows that both results have different intensity for the same thickness, because in this study, the roughness of burnt coal seam is considering. In the contrary, this matter was ignored in the previous analysis (an application using classical transmission line method). The results also prove that the roughness of targeted surface plays an important role in the analysis of surface scattering. Additionally, the result shows that hh- and vv-polarisations in L Band wave scattering have different intensity about –10 dB between each other. In the next section, the results are applied to estimate the thickness x of burnt coal seam in the study area.

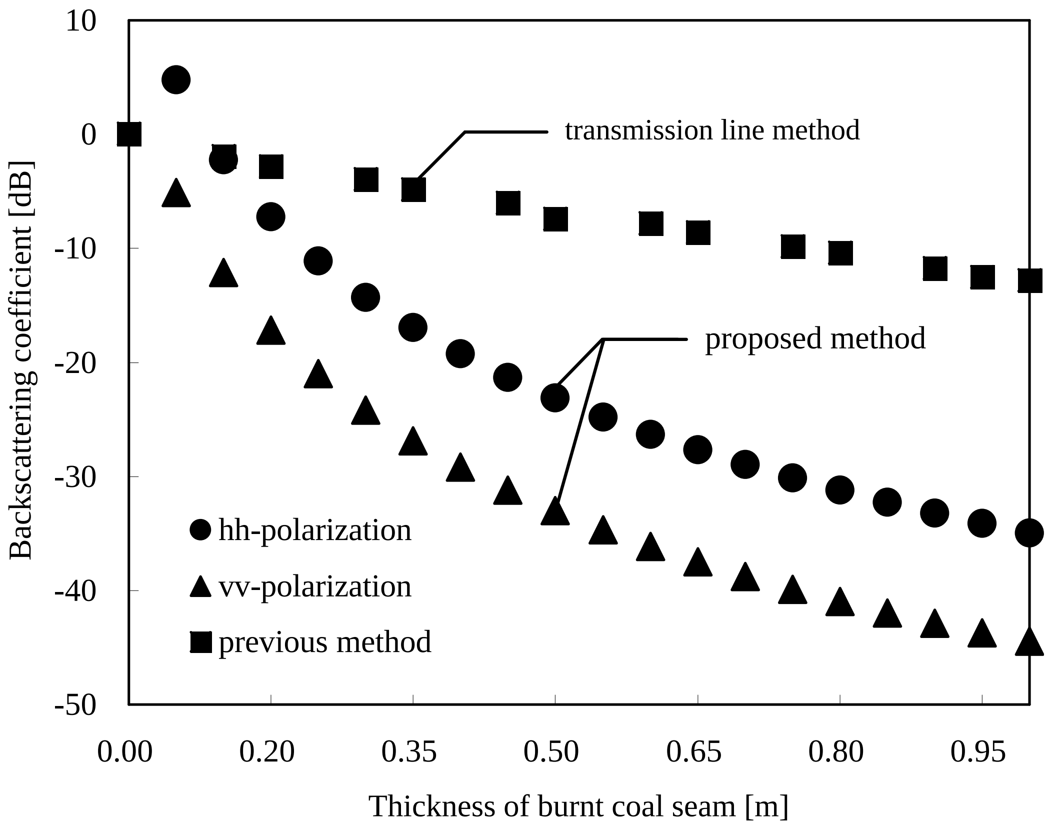

Figure 5.2. Relationship between the backscattering coefficient and the thickness of burnt coal seam.

5.4 Application

5.4.1 Study area

Study area is located 114°23′–114°45′E, 2°23′–2°47′S or at district of south Barito and Kapuas, central Borneo, Indonesia as shown in figure 4.2 (a) and (b). The altitude of this area is ranging from 9m to 14m above sea level. The Barito river and the Kapuas river are encircled this area, and vegetation type of this area is tropical forest. This area is mainly covered by peat (soil with about 60% coal content) and peat swamp (peat area with high water content). The climate on Borneo is wet all through the year with an average annual rainfall of around 3500mm to 4500mm, while the relative humidity varies between 70% and 90%.

5.4.2 Data processing

JERS-1 SAR data (path 95, row 305) that was acquired on 29 July 1997 (dry season and during fire events), see figure 4.11, is used to estimate the thickness of forest fire scars (burnt coal seam) in the study area. The data processing is done using the same process that was done in Chapter IV. The backscattering coefficient of each burnt coal seam class is shown in figure 4.13 and the value of it is depicted in table 4.1. By plotting the backscattering coefficient value into graph of 'proposed method' (hh-polarization) in figure 5.2, ξ of each class is acquired (see table 5.1), where the ξs in the study area are between 0.16 and 0.21m.

Table 5.1. Thickness of burnt coal seam in the study area

Class names	Backscattering coefficient (dB)	Burnt coal seam thickness ξ (m)	Standard deviation (m)
Burnt coal seam 1	-7.0	0.21	0.000
Burnt coal seam 2	-6.5	0.19	0.005
Burnt coal seam 3	-5.8	0.17	0.005
Burnt coal seam 4	-5.0	0.16	0.005

The result was confirmed by ground data (figure 4.2 (b)), where the A and B area in figure 4.13 have thickness of burnt coal seam is about 0 to 50m. Therefore, it agreed with the estimated result. The results show that the fires reached 0.21m in depth (thickness). This value is half of estimated value using previous method, although it is still in range of ground data (0 to 50m). This estimation error is causing by considering the surface roughness in the analysis process.

5.5 Conclusions

Numerical analysis was conducted to analyse the relationship between the backscattering coefficients s^{o} and burnt coal seam thickness ξ. This analysis result was confirmed by previous result that was derived using a classical transmission line method. Proposed result show about half value of previous estimated result. It causes by considering the roughness of burnt coal seam surface. This result was applied successfully in estimating the burnt coal seam thickness in central Borneo, Indonesia. The analysis result was confirmed by the ground data that was collected by ground survey done in 1995 to 1997, and it shows that fires reached 0.21m in depth (thickness). This is in good agreement with ground data. Further, application of this result can be used to estimate fire scars thickness, which is very important to extinguish forest fire effectively and accurately. This technique can be also applied to monitor the post-crisis management of fire events by estimating the damages suffer the soil and the carbon release into the atmosphere.

In this research, many parameters are still ignored. Hence, in the next research, shadowing and multiple scattering will be considered to obtain more real results.

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