Appendix J

Derivation of the Scattered Fields

in the Medium 1 on Air and Burnt Coal Seam Interface

From (5.97), the vector nature of ${}^{m}\boldsymbol{E}^{s}$ is characterized by

$$\begin{aligned}{}^{m}\boldsymbol{E}_{n}^{s} &\equiv \boldsymbol{E}^{s}/KI_{1} \\ &= \hat{\boldsymbol{n}}_{sm}^{t}\times\left(\hat{\boldsymbol{n}}_{1}\times\boldsymbol{E}-\hat{\boldsymbol{n}}_{sm}^{t}\times\left(h_{1}\hat{\boldsymbol{n}}_{1}\times\boldsymbol{H}\right)\right) \\ &= \hat{\boldsymbol{n}}_{sm}^{t}\times\left(\hat{\boldsymbol{n}}_{1}\times\boldsymbol{E}\right)+\left(h_{1}\hat{\boldsymbol{n}}_{1}\times\boldsymbol{H}\right)-\left(\hat{\boldsymbol{n}}_{sm}^{t}\cdot\left(h_{1}\hat{\boldsymbol{n}}_{1}\times\boldsymbol{H}\right)\right)\hat{\boldsymbol{n}}_{sm}^{t}\end{aligned} \tag{J.1}$$

The polarization factors are

$$\hat{\boldsymbol{h}}_{sm}\cdot{}^{m}\boldsymbol{E}_{n}^{s}=\hat{\boldsymbol{v}}_{sm}\cdot\left(\hat{\boldsymbol{n}}_{1}\times\boldsymbol{E}\right)+\hat{\boldsymbol{h}}_{sm}\cdot\left(h_{1}\hat{\boldsymbol{n}}_{1}\times\boldsymbol{H}\right) \tag{J.2}$$

$$\hat{\boldsymbol{v}}_{sm}\cdot{}^{m}\boldsymbol{E}_{n}^{s}=-\hat{\boldsymbol{h}}_{sm}\cdot\left(\hat{\boldsymbol{n}}_{1}\times\boldsymbol{E}\right)+\hat{\boldsymbol{v}}_{sm}\cdot\left(h_{1}\hat{\boldsymbol{n}}_{1}\times\boldsymbol{H}\right) \tag{J.3}$$

Under the stationary-phase approximation, $\hat{\boldsymbol{n}}_{1}$, $\hat{\boldsymbol{t}}$, $\hat{\boldsymbol{d}}$ can be expressed in terms of propagation vectors, $\hat{\boldsymbol{n}}_{sm}$ and $\hat{\boldsymbol{n}}_{im}$, as in Appendix G.

$$\hat{\boldsymbol{n}}_{1}=\frac{\left(\hat{\boldsymbol{x}}\bar{q}_{x}+\hat{\boldsymbol{y}}\bar{q}_{y}+\hat{\boldsymbol{z}}\bar{q}_{z}\right)\left|\bar{q}_{z}\right|}{\bar{q}_{z}\bar{q}}=\frac{\left(k_{1}\hat{\boldsymbol{n}}_{sm}-k_{1}\hat{\boldsymbol{n}}_{im}\right)\left|\bar{q}_{z}\right|}{\bar{q}\bar{q}_{z}} \tag{J.4}$$

where

$$\bar{q}^{2}=\bar{q}_{x}^{2}+\bar{q}_{y}^{2}+\bar{q}_{z}^{2}=k_{1}^{2}+k_{2}^{2}-2k_{1}k_{2}\left(\hat{\boldsymbol{n}}_{sm}\cdot\hat{\boldsymbol{n}}_{im}\right) \tag{J.5}$$

$$\hat{\boldsymbol{t}}=\frac{\left(\hat{\boldsymbol{n}}_{1}\times\hat{\boldsymbol{n}}_{sm}\right)\left|q_{z}\right|}{\bar{q}_{z}D_{2}^{m}} \tag{J.6}$$

$$D_{2}^{m}=\left|\hat{\boldsymbol{n}}_{im}\times\hat{\boldsymbol{n}}_{sm}\right|=\left|\hat{\boldsymbol{n}}_{im}\times\left(\hat{\boldsymbol{h}}_{sm}\times\hat{\boldsymbol{v}}_{sm}\right)\right|=\left|\left(\hat{\boldsymbol{n}}_{im}\cdot\hat{\boldsymbol{v}}_{sm}\right)^{2}+\left(\hat{\boldsymbol{n}}_{im}\cdot\hat{\boldsymbol{h}}_{sm}\right)^{2}\right|^{\frac{1}{2}} \tag{J.7}$$

$$\hat{\boldsymbol{d}}=\hat{\boldsymbol{n}}_{im}\times\hat{\boldsymbol{t}}=\left(\left(\hat{\boldsymbol{n}}_{im}\cdot\hat{\boldsymbol{n}}_{sm}\right)\hat{\boldsymbol{n}}_{im}-\hat{\boldsymbol{n}}_{sm}\right)\left|\bar{q}_{z}\right|/\left(\bar{q}_{z}D_{2}^{m}\right) \tag{J.8}$$

with these local coordinate vectors expressed in $\hat{\boldsymbol{n}}_{im}$ and $\hat{\boldsymbol{n}}_{sm}$, all other vector products can be expressed in $\hat{\boldsymbol{n}}_{im}$ and $\hat{\boldsymbol{n}}_{sm}$

$$\hat{\boldsymbol{n}}_1 \times \hat{\boldsymbol{t}} = \frac{\{k_2(\hat{\boldsymbol{n}}_{im} \cdot \hat{\boldsymbol{n}}_{sm}) - k_1\}\hat{\boldsymbol{n}}_{im} + \{k_1(\hat{\boldsymbol{n}}_{im} \cdot \hat{\boldsymbol{n}}_{sm}) - k_2\}\hat{\boldsymbol{n}}_{sm}}{\overline{q} D_2^m} \tag{J.9}$$

$$\hat{\boldsymbol{n}}_1 \cdot \hat{\boldsymbol{n}}_{sm} = -(k_1 - k_2(\hat{\boldsymbol{n}}_{im} \cdot \hat{\boldsymbol{n}}_{sm}))|\overline{q}_z| / \overline{q}\overline{q}_z = \hat{\boldsymbol{n}}_1 \cdot \hat{\boldsymbol{n}}_t \equiv \cos f_m \tag{J.10}$$

$$\hat{\boldsymbol{h}} \cdot \hat{\boldsymbol{t}} = (\hat{\boldsymbol{v}} \cdot \hat{\boldsymbol{n}}_{sm}) / \overline{D}_2^m \tag{J.11}$$

$$\hat{\boldsymbol{h}}_{\boldsymbol{sm}} \cdot \hat{\boldsymbol{t}} = -(\hat{\boldsymbol{v}}_{\boldsymbol{sm}} \cdot \hat{\boldsymbol{n}}_{im}) / \overline{D}_2^m \tag{J.12}$$

$$\hat{\boldsymbol{v}} \cdot \hat{\boldsymbol{t}} = -(\hat{\boldsymbol{h}} \cdot \hat{\boldsymbol{n}}_{sm}) / \overline{D}_2^m \tag{J.13}$$

$$\hat{\boldsymbol{v}}_{\boldsymbol{sm}} \cdot \hat{\boldsymbol{t}} = -(\hat{\boldsymbol{h}}_{\boldsymbol{sm}} \cdot \hat{\boldsymbol{n}}_{im}) / \overline{D}_2^m \tag{J.14}$$

$$\hat{\boldsymbol{h}} \cdot \hat{\boldsymbol{d}} = -(\hat{\boldsymbol{h}} \cdot \hat{\boldsymbol{n}}_{sm}) / \overline{D}_2^m \tag{J.15}$$

$$\hat{\boldsymbol{h}}_{\boldsymbol{sm}} \cdot \hat{\boldsymbol{d}} = (\hat{\boldsymbol{n}}_{im} \cdot \hat{\boldsymbol{n}}_{sm})(\hat{\boldsymbol{h}}_{\boldsymbol{sm}} \cdot \hat{\boldsymbol{n}}_{im}) / \overline{D}_2^m \tag{J.16}$$

$$\hat{\boldsymbol{v}} \cdot \hat{\boldsymbol{d}} = -(\hat{\boldsymbol{v}} \cdot \hat{\boldsymbol{n}}_{sm}) / \overline{D}_2^m \tag{J.17}$$

$$\hat{\boldsymbol{v}}_{\boldsymbol{sm}} \cdot \hat{\boldsymbol{d}} = (\hat{\boldsymbol{n}}_{im} \cdot \hat{\boldsymbol{n}}_{sm})(\hat{\boldsymbol{v}}_{\boldsymbol{sm}} \cdot \hat{\boldsymbol{n}}_{im}) / \overline{D}_2^m \tag{J.18}$$

where

$$D_2^m = \overline{q}_z D_2^m / |\overline{q}_z| \tag{J.19}$$

$$\hat{\boldsymbol{v}}_{sm} \cdot (\hat{\boldsymbol{n}}_1 \times \hat{\boldsymbol{t}}) = (\hat{\boldsymbol{v}}_{sm} \cdot \hat{\boldsymbol{n}}_{im})(k_2(\hat{\boldsymbol{n}}_{im} \cdot \hat{\boldsymbol{n}}_{sm}) - k_1) / (\overline{q} D_2^m) \tag{J.20}$$

$$\hat{\boldsymbol{h}}_{sm} \cdot (\hat{\boldsymbol{n}}_1 \times \hat{\boldsymbol{t}}) = (\hat{\boldsymbol{h}}_{sm} \cdot \hat{\boldsymbol{n}}_{im})(k_2(\hat{\boldsymbol{n}}_{im} \cdot \hat{\boldsymbol{n}}_{sm}) - k_1) / (\overline{q} D_1^m) \tag{J.21}$$

with the above vector identities (J.2) and (J.3) can be expressed in $\hat{\boldsymbol{n}}_{im}$ and $\hat{\boldsymbol{n}}_{sm}$. For horizontally polarized incident wave

$$\hat{\boldsymbol{v}}_{sm}\cdot(\hat{\boldsymbol{n}}_1\times\boldsymbol{E})=\left(T_\perp(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{t}})(\hat{\boldsymbol{v}}_{sm}\cdot\hat{\boldsymbol{n}}_1\times\hat{\boldsymbol{t}})-T_\parallel(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{d}})(\hat{\boldsymbol{n}}_1\cdot\hat{\boldsymbol{n}}_t)(\hat{\boldsymbol{v}}_{sm}\cdot\hat{\boldsymbol{t}})h_1/h_2\right)E_o$$
$$=-\{k_1-k_2(\hat{\boldsymbol{n}}_{im}\cdot\hat{\boldsymbol{n}}_{sm})\}E_o\times\frac{T_\perp(\hat{\boldsymbol{v}}\cdot\hat{\boldsymbol{n}}_{sm})(\hat{\boldsymbol{v}}_{sm}\cdot\hat{\boldsymbol{n}}_{im})+T_\parallel(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{n}}_{sm})(\hat{\boldsymbol{h}}_{sm}\cdot\hat{\boldsymbol{n}}_{im})h_1/h_2}{\overline{q}D_2^m\overline{D}_2^m} \quad \text{(J.22)}$$

$$\hat{\boldsymbol{h}}_{sm}\cdot(h_1\hat{\boldsymbol{n}}_1\times\boldsymbol{H})=-\left(T_\perp(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{t}})(\hat{\boldsymbol{n}}_2\cdot\hat{\boldsymbol{n}}_t)(\hat{\boldsymbol{h}}_{sm}\cdot\hat{\boldsymbol{t}})+T_\parallel(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{d}})(\hat{\boldsymbol{h}}_{sm}\cdot\hat{\boldsymbol{n}}_1\times\hat{\boldsymbol{t}})h_1/h_2\right)E_o$$
$$=\text{(J.22)} \quad \text{(J.23)}$$

$$\hat{\boldsymbol{v}}_{sm}\cdot(h_1\hat{\boldsymbol{n}}_1\times\boldsymbol{H})=-\left(T_\perp(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{t}})(\hat{\boldsymbol{n}}_1\cdot\hat{\boldsymbol{n}}_t)(\hat{\boldsymbol{v}}_{sm}\cdot\hat{\boldsymbol{t}})+T_\parallel(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{d}})(\hat{\boldsymbol{v}}_{sm}\cdot\hat{\boldsymbol{n}}_1\times\hat{\boldsymbol{t}})h_1/h_2\right)E_o$$
$$=\{k_1-k_2(\hat{\boldsymbol{n}}_{im}\cdot\hat{\boldsymbol{n}}_{sm})\}E_o\times\frac{T_\perp(\hat{\boldsymbol{v}}\cdot\hat{\boldsymbol{n}}_{sm})(\hat{\boldsymbol{h}}_{sm}\cdot\hat{\boldsymbol{n}}_{im})-T_\parallel(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{n}}_{sm})(\hat{\boldsymbol{v}}_{sm}\cdot\hat{\boldsymbol{n}}_{im})h_1/h_2}{\overline{q}D_2^m\overline{D}_2^m} \quad \text{(J.24)}$$

$$\hat{\boldsymbol{h}}_{sm}\cdot(\hat{\boldsymbol{n}}_1\times\boldsymbol{E})=\left(T_\perp(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{t}})(\hat{\boldsymbol{h}}_{sm}\cdot\hat{\boldsymbol{n}}_1\times\hat{\boldsymbol{t}})-T_\parallel(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{d}})(\hat{\boldsymbol{n}}_1\cdot\hat{\boldsymbol{n}}_t)(\hat{\boldsymbol{h}}_{sm}\cdot\hat{\boldsymbol{t}})h_1/h_2\right)E_o$$
$$=-(\text{J.24}) \quad \text{(J.25)}$$

By substituting (J.22) through (J.25) into (J.2) and (J.3), we obtain

$$ {}^mE_{hh}^s=KI_m\left(\hat{\boldsymbol{h}}_{sm}\cdot{}^m\boldsymbol{E}_n^s\right)$$
$$=-M_m\left\{T_\perp(\hat{\boldsymbol{v}}\cdot\hat{\boldsymbol{n}}_{sm})(\hat{\boldsymbol{v}}_{sm}\cdot\hat{\boldsymbol{n}}_{im})+T_\parallel(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{n}}_{sm})(\hat{\boldsymbol{h}}_{sm}\cdot\hat{\boldsymbol{n}}_{im})h_1/h_2\right\} \quad \text{(J.26)}$$

$$ {}^mE_{vh}^s=KI_m\left(\hat{\boldsymbol{v}}_{sm}\cdot{}^m\boldsymbol{E}_n^s\right)$$
$$=M_m\left\{T_\perp(\hat{\boldsymbol{v}}_{sm}\cdot\hat{\boldsymbol{n}}_{im})(\hat{\boldsymbol{h}}\cdot\hat{n}_{sm})-T_\parallel(\hat{\boldsymbol{h}}_{sm}\cdot\hat{\boldsymbol{n}}_{im})(\hat{\boldsymbol{v}}\cdot\hat{\boldsymbol{n}}_{sm})h_1/h_2\right\} \quad \text{(J.27)}$$

where $M_m=2KI_mE_o(k_1-k_2(\hat{\boldsymbol{n}}_{im}\cdot\hat{\boldsymbol{n}}_{sm}))/(\overline{q}D_2^m\overline{D}_2^m)$. For a vertically polarized incident wave, ${}^mE_{vv}^s$ and ${}^mE_{hv}^s$ can be obtained from (J.26) and (J.27) respectively by interchanging $\hat{\boldsymbol{v}}$ with $\hat{\boldsymbol{h}}$, and $\hat{\boldsymbol{v}}_{sm}$ with $\hat{\boldsymbol{h}}_{sm}$

$$ {}^mE_{vv}^s=-M_m\left\{T_\perp(\hat{\boldsymbol{h}}_{sm}\cdot\hat{\boldsymbol{n}}_{im})(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{n}}_{sm})+T_\parallel(\hat{\boldsymbol{v}}_{sm}\cdot\hat{\boldsymbol{n}}_{im})(\hat{\boldsymbol{v}}\cdot\hat{\boldsymbol{n}}_{sm})h_1/h_2\right\} \quad \text{(J.28)}$$

$$ {}^mE_{hv}^s=M_m\left\{T_\perp(\hat{\boldsymbol{v}}_{sm}\cdot\hat{\boldsymbol{n}}_{im})(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{n}}_{sm})-T_\parallel(\hat{\boldsymbol{h}}_{sm}\cdot\hat{\boldsymbol{n}}_{im})(\hat{\boldsymbol{v}}\cdot\hat{\boldsymbol{n}}_{sm})h_1/h_2\right\} \quad \text{(J.29)}$$

The dot products in the field expressions are given by

$$\hat{\boldsymbol{v}}_{sm}\cdot\hat{\boldsymbol{n}}_{im}=-\sin q_m\cos q_{sm}^t\cos(f_{sm}-f_m)+\cos q_m\sin q_{sm}^t \quad \text{(J.30)}$$

$$\hat{\boldsymbol{v}}\cdot\hat{\boldsymbol{n}}_{sm}=-\cos q_m\sin q_{sm}^t\cos(f_{sm}-f_m)+\sin q_m\cos q_{sm}^t \quad \text{(J.31)}$$

$$\hat{\boldsymbol{h}}_{sm} \cdot \hat{\boldsymbol{n}}_{im} = -\sin q_m \sin\left(f_{sm} - f_m\right) \tag{J.32}$$

$$\hat{\boldsymbol{h}} \cdot \hat{\boldsymbol{n}}_{sm} = \sin q_{sm}^{t} \sin\left(f_{sm} - f_m\right) \tag{J.33}$$