Appendix I

Derivation of the Scattered Fields

in the Medium 2 on Air and Burnt Coal Seam Interface

From (5.79), the vector nature of ${}^{m}\boldsymbol{E}^{s}$ is characterized by

$$
\begin{aligned}
{}^{m}\boldsymbol{E}_{n}^{s} &\equiv {}^{m}\boldsymbol{E}^{s}/KI_{m} \\
&= \hat{\boldsymbol{n}}_{sm}\times\left(\hat{\boldsymbol{n}}_{2}\times\boldsymbol{E}-h_{2}\hat{\boldsymbol{n}}_{sm}\times\left(\hat{\boldsymbol{n}}_{2}\times\boldsymbol{H}\right)\right) \\
&= \hat{\boldsymbol{n}}_{sm}\times\left(\hat{\boldsymbol{n}}_{2}\times\boldsymbol{E}\right)+\left(h_{2}\hat{\boldsymbol{n}}_{2}\times\boldsymbol{H}\right)-\left(\hat{\boldsymbol{n}}_{sm}\cdot\left(h_{2}\hat{\boldsymbol{n}}_{2}\times\boldsymbol{H}\right)\right)\hat{\boldsymbol{n}}_{sm}
\end{aligned}
\tag{I.1}
$$

the polarization factors which characterize different polarisation states are

$$\hat{\boldsymbol{h}}_{sm}\cdot{}^{m}\boldsymbol{E}_{n}^{s}=\hat{\boldsymbol{v}}_{sm}\cdot\left(\hat{\boldsymbol{n}}_{2}\times\boldsymbol{E}\right)+\hat{\boldsymbol{h}}_{sm}\cdot\left(h_{2}\hat{\boldsymbol{n}}_{2}\times\boldsymbol{H}\right) \tag{I.2}$$

$$\hat{\boldsymbol{v}}_{sm}\cdot{}^{m}\boldsymbol{E}_{n}^{s}=\hat{\boldsymbol{v}}_{sm}\cdot\left(h_{2}\hat{\boldsymbol{n}}_{2}\times\boldsymbol{H}\right)-\hat{\boldsymbol{h}}_{sm}\cdot\left(\hat{\boldsymbol{n}}_{2}\times\boldsymbol{E}\right) \tag{I.3}$$

Under the stationary-phase approximation, $\hat{\boldsymbol{n}}_{2}$, $\hat{\boldsymbol{t}}$, $\hat{\boldsymbol{d}}$ can be expressed in terms of propagation vectors, $\hat{\boldsymbol{n}}_{sm}$ and $\hat{\boldsymbol{n}}_{im}$, as

$$
\begin{aligned}
\hat{\boldsymbol{n}}_{2} &= \frac{-\hat{\boldsymbol{x}}Z_{x}-\hat{\boldsymbol{y}}Z_{y}+\hat{\boldsymbol{z}}}{\left(1+Z_{x}^{2}+Z_{y}^{2}\right)^{\frac{1}{2}}}=\frac{\hat{\boldsymbol{x}}q_{x}+\hat{\boldsymbol{y}}q_{y}+\hat{\boldsymbol{z}}q_{z}\left|q_{z}\right|}{q_{z}q} \\
&= \frac{k_{2}\left(\hat{\boldsymbol{n}}_{sm}-\hat{\boldsymbol{n}}_{im}\right)\left|q_{z}\right|}{q_{z}q}
\end{aligned}
\tag{I.4}
$$

where $q^{2}=q_{x}^{2}+q_{y}^{2}+q_{z}^{2}=2k_{2}^{2}\left(1-\left(\hat{\boldsymbol{n}}_{sm}\cdot\hat{\boldsymbol{n}}_{im}\right)\right)$

$$
\begin{aligned}
\hat{\boldsymbol{t}} &= \frac{\hat{\boldsymbol{n}}_{im}\times\hat{\boldsymbol{n}}_{2}}{\left|\hat{\boldsymbol{n}}_{im}\times\hat{\boldsymbol{n}}_{2}\right|}=\frac{\left(\hat{\boldsymbol{n}}_{2}\times\hat{\boldsymbol{n}}_{sm}\right)\left|q_{z}\right|}{q_{z}D_{1}^{m}} \\
&= \frac{\left(\hat{\boldsymbol{x}}q_{y}\cos q_{o}-\hat{\boldsymbol{y}}\left(q_{x}\cos q_{o}+q_{z}\sin q_{o}\right)+\hat{\boldsymbol{z}}q_{y}\sin q_{o}\right)}{q_{z}k_{2}^{2}D_{1}^{m}}
\end{aligned}
\tag{I.5}
$$

where

$$D_1^m = \left|\hat{\boldsymbol{n}}_{im} \times \hat{\boldsymbol{n}}_{sm}\right| = \left|\left(\hat{\boldsymbol{n}}_{im} \cdot \hat{\boldsymbol{v}}_{sm}\right)^2 + \left(\hat{\boldsymbol{n}}_{im} \cdot \hat{\boldsymbol{h}}_{sm}\right)^2\right|^{\frac{1}{2}} \tag{I.6}$$

and

$$\begin{aligned}\hat{\boldsymbol{d}} &= \hat{\boldsymbol{n}} \times \hat{\boldsymbol{t}} = \left(\left(\hat{\boldsymbol{n}}_{im} \cdot \hat{\boldsymbol{n}}_{sm}\right)\hat{\boldsymbol{n}}_{im} - \hat{\boldsymbol{n}}_{sm}\right)\left|q_z\right| \Big/ \left(q_z D_1^m\right) \\ &= \frac{-\left(\hat{\boldsymbol{x}}\cos\boldsymbol{q}_m\left(q_x\cos\boldsymbol{q}_m + q_z\sin\boldsymbol{q}_m\right) + \hat{\boldsymbol{y}}q_y + \hat{\boldsymbol{z}}\sin\boldsymbol{q}_m\left(q_x\cos\boldsymbol{q}_m + q_z\sin\boldsymbol{q}_m\right)\right)}{q_z k_2^2 D_1^m}\end{aligned} \tag{I.7}$$

with these local coordinate vectors expressed in $\hat{\boldsymbol{n}}_{im}$ and $\hat{\boldsymbol{n}}_{sm}$, all other vector products can be expressed in $\hat{\boldsymbol{n}}_{im}$ and $\hat{\boldsymbol{n}}_{sm}$

$$\hat{\boldsymbol{n}}_2 \times \hat{\boldsymbol{t}} = \left(\hat{\boldsymbol{n}}_{im} + \hat{\boldsymbol{n}}_{sm}\right)\left(1 - \left(\hat{\boldsymbol{n}}_{im} \cdot \hat{\boldsymbol{n}}_{sm}\right)\right)k_2 \Big/ \left(q D_1^m\right) \tag{I.8}$$

$$\hat{\boldsymbol{n}}_2 \cdot \hat{\boldsymbol{n}}_{im} = -k_2\left\{1 - \left(\hat{\boldsymbol{n}}_{im} \cdot \hat{\boldsymbol{n}}_{sm}\right)\right\}\left|q_z\right| \big/ \left(q_z q\right) = -q\left|q_z\right| \big/ \left(2k_2 q_z\right) = -\cos\boldsymbol{q}_m \tag{I.9}$$

$$\hat{\boldsymbol{h}} \cdot \hat{\boldsymbol{t}} = \left(\hat{\boldsymbol{v}} \cdot \hat{\boldsymbol{n}}_{sm}\right) \big/ \overline{D}_1^m \tag{I.10}$$

$$\hat{\boldsymbol{h}}_{\boldsymbol{sm}} \cdot \hat{\boldsymbol{t}} = -\left(\hat{\boldsymbol{v}}_{\boldsymbol{sm}} \cdot \hat{\boldsymbol{n}}_{im}\right) \big/ \overline{D}_1^m \tag{I.11}$$

$$\hat{\boldsymbol{v}} \cdot \hat{\boldsymbol{t}} = -\left(\hat{\boldsymbol{h}} \cdot \hat{\boldsymbol{n}}_{sm}\right) \big/ \overline{D}_1^m \tag{I.12}$$

$$\hat{\boldsymbol{v}}_{\boldsymbol{sm}} \cdot \hat{\boldsymbol{t}} = -\left(\hat{\boldsymbol{h}}_{\boldsymbol{sm}} \cdot \hat{\boldsymbol{n}}_{im}\right) \big/ \overline{D}_1^m \tag{I.13}$$

$$\hat{\boldsymbol{h}} \cdot \hat{\boldsymbol{d}} = -\left(\hat{\boldsymbol{h}} \cdot \hat{\boldsymbol{n}}_{sm}\right) \big/ \overline{D}_1^m \tag{I.14}$$

$$\hat{\boldsymbol{h}}_{\boldsymbol{sm}} \cdot \hat{\boldsymbol{d}} = \left(\hat{\boldsymbol{h}}_{\boldsymbol{sm}} \cdot \hat{\boldsymbol{n}}_{im}\right)\left(\hat{\boldsymbol{n}}_{sm} \cdot \hat{\boldsymbol{n}}_{im}\right) \big/ \overline{D}_1^m \tag{I.15}$$

$$\hat{\boldsymbol{v}} \cdot \hat{\boldsymbol{d}} = -\left(\hat{\boldsymbol{v}} \cdot \hat{\boldsymbol{n}}_{sm}\right) \big/ \overline{D}_1^m \tag{I.16}$$

$$\hat{\boldsymbol{v}}_{\boldsymbol{sm}} \cdot \hat{\boldsymbol{d}} = \left(\hat{\boldsymbol{v}}_{\boldsymbol{sm}} \cdot \hat{\boldsymbol{n}}_{im}\right)\left(\hat{\boldsymbol{n}}_{sm} \cdot \hat{\boldsymbol{n}}_{im}\right) \big/ \overline{D}_1^m \tag{I.17}$$

where

$$D_1^m = q_z D_1^m \big/ \left|q_z\right| \tag{I.18}$$

$$\hat{\boldsymbol{v}}_{sm}\cdot\left(\hat{\boldsymbol{n}}_2\times\hat{\boldsymbol{t}}\right)=\left(\hat{\boldsymbol{v}}_{sm}\cdot\hat{\boldsymbol{n}}_{im}\right)\left(1-\left(\hat{\boldsymbol{n}}_{im}\cdot\hat{\boldsymbol{n}}_{sm}\right)\right)k_2\Big/\left(qD_1^m\right)$$
$$=\left(\hat{\boldsymbol{v}}_{sm}\cdot\hat{\boldsymbol{n}}_{im}\right)q\Big/\left(2k_2D_1^m\right) \tag{I.19}$$

$$\hat{\boldsymbol{h}}_{sm}\cdot\left(\hat{\boldsymbol{n}}_2\times\hat{\boldsymbol{t}}\right)=\left(\hat{\boldsymbol{h}}_{sm}\cdot\hat{\boldsymbol{n}}_{im}\right)\left(1-\left(\hat{\boldsymbol{n}}_{im}\cdot\hat{\boldsymbol{n}}_{sm}\right)\right)k_2\Big/\left(qD_1^m\right)$$
$$=\left(\hat{\boldsymbol{h}}_{sm}\cdot\hat{\boldsymbol{n}}_{im}\right)q\Big/\left(2k_2D_1^m\right) \tag{I.20}$$

with the above vector identities (I.2) and (I.3) can be expressed in $\hat{\boldsymbol{n}}_{im}$ and $\hat{\boldsymbol{n}}_{sm}$. For horizontally polarized incident wave

$$\hat{\boldsymbol{v}}_{sm}\cdot\left(\hat{\boldsymbol{n}}_2\times\boldsymbol{E}\right)=\left(\left(1+R_\perp\right)\left(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{t}}\right)\left(\hat{\boldsymbol{v}}_{sm}\cdot\hat{\boldsymbol{n}}_2\times\hat{\boldsymbol{t}}\right)-\left(1-R_\parallel\right)\left(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{d}}\right)\left(\hat{\boldsymbol{n}}_2\cdot\hat{\boldsymbol{n}}_{im}\right)\left(\hat{\boldsymbol{v}}_{sm}\cdot\hat{\boldsymbol{t}}\right)\right)E_o$$
$$=\frac{qE_o}{2k_2D_1^m\overline{D}_1^m}\left\{\left(1+R_\perp\right)\left(\hat{\boldsymbol{v}}\cdot\hat{\boldsymbol{n}}_{sm}\right)\left(\hat{\boldsymbol{v}}_{sm}\cdot\hat{\boldsymbol{n}}_{im}\right)-\left(1-R_\parallel\right)\left(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{n}}_{sm}\right)\left(\hat{\boldsymbol{h}}_{sm}\cdot\hat{\boldsymbol{n}}_{im}\right)\right\} \tag{I.21}$$

$$\hat{\boldsymbol{h}}_{sm}\cdot\left(h_{\boldsymbol{1}}\hat{\boldsymbol{n}}_2\times\boldsymbol{H}\right)=-\left(\left(1-R_\perp\right)\left(\hat{\boldsymbol{n}}_2\cdot\hat{\boldsymbol{n}}_{im}\right)\left(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{t}}\right)\left(\hat{\boldsymbol{h}}_{sm}\cdot\hat{\boldsymbol{t}}\right)+\left(1+R_\parallel\right)\left(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{d}}\right)\left(\hat{\boldsymbol{h}}_{sm}\cdot\hat{\boldsymbol{n}}_2\times\hat{\boldsymbol{t}}\right)\right)E_o$$
$$=\frac{qE_o}{2k_2D_1^m\overline{D}_1^m}\left\{-\left(1-R_\perp\right)\left(\hat{\boldsymbol{v}}_{\boldsymbol{sm}}\cdot\hat{\boldsymbol{n}}_{im}\right)\left(\hat{\boldsymbol{v}}\cdot\hat{\boldsymbol{n}}_{sm}\right)+\left(1+R_\parallel\right)\left(\hat{\boldsymbol{h}}_{\boldsymbol{sm}}\cdot\hat{\boldsymbol{n}}_{im}\right)\left(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{n}}_{sm}\right)\right\} \tag{I.22}$$

Equation (5.85) for ${}^mE_{hh}^s$ can be found from (I.20) and (I.21), since

$$\hat{\boldsymbol{h}}_{sm}\cdot\boldsymbol{E}^s=KI_m\left\{(\text{I.20})+(\text{I.21})\right\}$$
$$=M_m\left\{R_\parallel\left(\hat{\boldsymbol{h}}_{sm}\cdot\hat{\boldsymbol{n}}_{im}\right)\left(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{n}}_{sm}\right)+R_\perp\left(\hat{\boldsymbol{v}}_{sm}\cdot\hat{\boldsymbol{n}}_{im}\right)\left(\hat{\boldsymbol{v}}\cdot\hat{\boldsymbol{n}}_{sm}\right)\right\} \tag{I.23}$$

where

$$M_m=KI_mE_oq\Big/\left(k_2D_1^m\overline{D}_1^m\right) \tag{I.24}$$

To compute ${}^oE_{vh}^s$, we need the following vector products:

$$\hat{\boldsymbol{v}}_{sm}\cdot\left(h_{\boldsymbol{1}}\hat{\boldsymbol{n}}_2\times\boldsymbol{H}\right)=-\left(\left(1-R_\perp\right)\left(\hat{\boldsymbol{n}}_2\cdot\hat{\boldsymbol{n}}_{im}\right)\left(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{t}}\right)\left(\hat{\boldsymbol{v}}_{sm}\cdot\hat{\boldsymbol{t}}\right)+\left(1+R_\parallel\right)\left(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{d}}\right)\left(\hat{\boldsymbol{v}}_{sm}\cdot\hat{\boldsymbol{n}}_2\times\hat{\boldsymbol{t}}\right)\right)E_o$$
$$=\frac{qE_o}{2k_2D_1^m\overline{D}_1^m}\left\{\left(1-R_\perp\right)\left(\hat{\boldsymbol{h}}_{\boldsymbol{sm}}\cdot\hat{\boldsymbol{n}}_{im}\right)\left(\hat{\boldsymbol{v}}\cdot\hat{\boldsymbol{n}}_{sm}\right)+\left(1+R_\parallel\right)\left(\hat{\boldsymbol{v}}_{\boldsymbol{sm}}\cdot\hat{\boldsymbol{n}}_{im}\right)\left(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{n}}_{sm}\right)\right\} \tag{I.25}$$

$$\hat{\boldsymbol{h}}_{sm}\cdot\left(\hat{\boldsymbol{n}}_2\times\boldsymbol{E}\right)=\left(\left(1+R_\perp\right)\left(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{t}}\right)\left(\hat{\boldsymbol{h}}_{sm}\cdot\hat{\boldsymbol{n}}_2\times\hat{\boldsymbol{t}}\right)-\left(1-R_\parallel\right)\left(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{d}}\right)\left(\hat{\boldsymbol{n}}_2\cdot\hat{\boldsymbol{n}}_{im}\right)\left(\hat{\boldsymbol{h}}_{sm}\cdot\hat{\boldsymbol{t}}\right)\right)E_o$$
$$=\frac{qE_o}{2k_2D_1^m\overline{D}_1^m}\left\{\left(1+R_\perp\right)\left(\hat{\boldsymbol{v}}\cdot\hat{\boldsymbol{n}}_{sm}\right)\left(\hat{\boldsymbol{h}}_{sm}\cdot\hat{\boldsymbol{n}}_{im}\right)+\left(1-R_\parallel\right)\left(\hat{\boldsymbol{h}}\cdot\hat{\boldsymbol{n}}_{sm}\right)\left(\hat{\boldsymbol{v}}_{sm}\cdot\hat{\boldsymbol{n}}_{im}\right)\right\} \tag{I.26}$$

In view of (I.3)

$$
\begin{aligned}
{}^{m}E_{vh}^{s} &= \hat{\boldsymbol{v}}_{sm} \cdot \boldsymbol{E}^{s} \\
&= M_1 \left\{ R_{\parallel} \left(\hat{\boldsymbol{v}}_{sm} \cdot \hat{\boldsymbol{n}}_{im} \right) \left(\hat{\boldsymbol{h}} \cdot \hat{n}_{sm} \right) - R_{\perp} \left(\hat{\boldsymbol{h}}_{sm} \cdot \hat{\boldsymbol{n}}_{im} \right) \left(\hat{\boldsymbol{v}} \cdot \hat{\boldsymbol{n}}_{sm} \right) \right\}
\end{aligned}
\tag{I.27}
$$

The above equation is the same as (5.86). For vertically polarized incident wave, ${}^{o}E_{vv}^{s}$ can be obtained from (G.22) by interchanging $\hat{\boldsymbol{v}}$ and $\hat{\boldsymbol{h}}$ as well as $\hat{\boldsymbol{v}}_{\boldsymbol{sm}}$ and $\hat{\boldsymbol{h}}_{\boldsymbol{sm}}$:

$$
{}^{m}E_{vv}^{s} = M_1 \left\{ R_{\parallel} \left(\hat{\boldsymbol{v}}_{sm} \cdot \hat{\boldsymbol{n}}_{im} \right) \left(\hat{\boldsymbol{v}} \cdot \hat{\boldsymbol{n}}_{sm} \right) + R_{\perp} \left(\hat{\boldsymbol{h}}_{sm} \cdot \hat{\boldsymbol{n}}_{im} \right) \left(\hat{\boldsymbol{h}} \cdot \hat{\boldsymbol{n}}_{sm} \right) \right\}
\tag{I.28}
$$

Similarly, ${}^{o}E_{hv}^{s}$ can be found from (G.26) with the same interchanges:

$$
{}^{m}E_{hv}^{s} = M_1 \left\{ R_{\parallel} \left(\hat{\boldsymbol{h}}_{sm} \cdot \hat{\boldsymbol{n}}_{im} \right) \left(\hat{\boldsymbol{v}} \cdot \hat{\boldsymbol{n}}_{sm} \right) - R_{\perp} \left(\hat{\boldsymbol{v}}_{sm} \cdot \hat{\boldsymbol{n}}_{im} \right) \left(\hat{\boldsymbol{h}} \cdot \hat{\boldsymbol{n}}_{sm} \right) \right\}
\tag{I.29}
$$

in (G.22) through (G.28)

$$
R_{\parallel} = \left(k_1 \cos q_m - k_2 \cos f_m \right) / \left(k_1 \cos q_m + k_2 \cos f_m \right)
\tag{I.30}
$$

$$
R_{\perp} = \left(k_2 \cos q_m - k_1 \cos f_m \right) / \left(k_2 \cos q_m + k_1 \cos f_m \right)
\tag{I.31}
$$

where f_m is the local angle of transmission. The dot products in the field expressions are given below:

$$
\hat{\boldsymbol{v}}_{sm} \cdot \hat{\boldsymbol{n}}_{im} = \sin q_m \cos q_{sm} \cos\left(f_{sm} - f_m \right) + \cos q_m \sin q_{sm}
\tag{I.32}
$$

$$
\hat{\boldsymbol{v}} \cdot \hat{\boldsymbol{n}}_{sm} = \cos q_m \sin q_{sm} \cos\left(f_{sm} - f_m \right) + \sin q_m \cos q_{sm}
\tag{I.33}
$$

$$
\hat{\boldsymbol{h}}_{sm} \cdot \hat{\boldsymbol{n}}_{im} = -\sin q_m \sin\left(f_{sm} - f_m \right)
\tag{I.34}
$$

$$
\hat{\boldsymbol{h}} \cdot \hat{\boldsymbol{n}}_{sm} = \sin q_{sm} \sin\left(f_{sm} - f_m \right)
\tag{I.35}
$$