Appendix E

Wave Analysis in Cylindrical Coordinate System (TM mode)

Electric field of incident wave $E_z^I = E_o^I \exp(jk_o x)$ could be transformed into cylindrical coordinate system using equation

$$e^{jx\sin q} = \sum_{m=-\infty}^{\infty} J_m(x) e^{jmq} \qquad \text{(E.1).}$$

where J_m and E_o^I are Bessel function and intensity of the incident wave, respectively. Consequently, we will obtain the equation

$$\begin{aligned} e^{jk_o x} &= e^{jk_o r\cos f} = \sum_{m=-\infty}^{\infty} J_m(k_o r) e^{jm\left(\frac{p}{2}-f\right)} \\ &= J_o(k_o r) + \sum_{m=1}^{\infty}\left(J_m(k_o r) e^{j\frac{mp}{2}} e^{-jmf} + J_{-m}(k_o r) e^{-j\frac{mp}{2}} e^{jmf}\right) \end{aligned}$$

By using the relationship of $J_{-m}(k_o r) = (-1)^m J_m(k_o r)$, the above equation will be

$$\begin{aligned} e^{jk_o x} &= J_o(k_o r) + \sum_{m=1}^{\infty}\left(J_m(k_o r) e^{jmf} + (-1)^m J_m(k_o r) j^{-2m} e^{-jmf}\right) j^m \\ &= J_o(k_o r) + \sum_{m=1}^{\infty} 2 J_m(k_o r) j^m \cos mf \\ &= \sum_{m=0}^{\infty} U_m J_m(k_o r) j^m \cos mf \qquad \text{(E.2).} \end{aligned}$$

where

$$U_m = \begin{cases} 1 & (m = 0) \\ 2 & (m > 0) \end{cases}$$

Then the scattered electric in each media was defined as

$$E_z^S = E_o^I \sum_{m=0}^{\infty} b_m H_m^{(2)}(k_o r) \cos mf \qquad \text{(E.3)}$$

$$E_z^m = E_o^I \sum_{m=0}^{\infty} \left\{a_m J_m(kr) + c_m N_m(kr)\right\} \cos mf \qquad \text{(E.4)}$$

By substituting (E.3) and (E.4) into under mentioned Maxwell equation

$$H_f = \frac{1}{jwm}\frac{\partial E_z}{\partial r} \quad \text{(E.5)}$$

the magnetic fields in each media were obtained

$$H_f^I = \frac{k_o E_o^I}{jwm_o}\sum_{m=0}^{\infty} U_m J'_m(k_o r) j^m \cos mf \quad \text{(E.6)}$$

$$H_f^S = \frac{k_o E_o^I}{jwm_o}\sum_{m=0}^{\infty} b_m H_m^{(2)\prime}(k_o r)\cos mf \quad \text{(E.7)}$$

$$H_f^m = \frac{kE_o^I}{jwm}\sum_{m=0}^{\infty}\{a_m J'_m(kr) + c_m N'_m(kr)\}\cos mf \quad \text{(E.8)}$$

where J'_m, H'_m, and N'_m are differential Bessel, Hankel and Neumann functions, respectively. And a_m, b_m, and c_m are constants that obtained by substituting (E.2) to (E.4) and (E.6) to (E.8) into boundary conditions.