Appendix C

Finite Difference Time Domain Method (TE mode)

Maxwell's equations in homogeneous and non-dispersive medium are defined as

$$\frac{\partial \boldsymbol{E}}{\partial t} = -\frac{s}{e}\boldsymbol{E} + \frac{1}{e}\nabla \times \boldsymbol{H} \qquad \text{(C.1)}$$

$$\frac{\partial \boldsymbol{H}}{\partial t} = -\frac{1}{m}\nabla \times \boldsymbol{E} \qquad \text{(C.2)}$$

Where $\boldsymbol{E}$ and $\boldsymbol{H}$ are total electric and magnetic fields in simulation space, respectively. e, m, and s are dielectric constant, permeability, and conductivity of simulation media, respectively. In this analysis, incident wave is propagating in free space before reach the burnt coal seam. By considering the Maxwell's equations in free space, the incident electric and magnetic fields are obtained as

$$\frac{\partial \boldsymbol{E}^{\boldsymbol{I}}}{\partial t} = \frac{1}{e_o}\nabla \times \boldsymbol{H}^{\boldsymbol{I}} \qquad \text{(C.3)}$$

$$\frac{\partial \boldsymbol{H}^{\boldsymbol{I}}}{\partial t} = -\frac{1}{m}\nabla \times \boldsymbol{E}^{\boldsymbol{I}} \qquad \text{(C.4)}$$

In this research, scattered fields will be analysed, therefore, incident and scattered fields must be separated. Hence total fields are shown by

$$\boldsymbol{E} = \boldsymbol{E}^{I} + \boldsymbol{E}^{S} \qquad \text{(C.5)}$$

$$\boldsymbol{H} = \boldsymbol{H}^{I} + \boldsymbol{H}^{S} \qquad \text{(C.6)}$$

By substituting (C.3) to (C.4) and (C.5) to (C.6) into (C.1) to (C.2), the next equations are obtained

$$\frac{\partial \boldsymbol{E}^S}{\partial t} = -\frac{s}{e}\boldsymbol{E}^S + \frac{1}{e}\nabla\times\boldsymbol{H}^S - \frac{s}{e}\boldsymbol{E}^I - \frac{e-e_o}{e}\frac{\partial \boldsymbol{E}^I}{\partial t} \tag{C.7}$$

$$\frac{\partial \boldsymbol{H}^S}{\partial t} = -\frac{1}{m}\nabla\times\boldsymbol{E}^S - \frac{m-m_o}{m}\frac{\partial \boldsymbol{H}^I}{\partial t} \tag{C.8}$$

To obtain the calculation stability of incident and scattered electric field component in $t=\left(n-\frac{1}{2}\right)\Delta t$, there are sampled as

$$\boldsymbol{E}^S\Big|_{t=\left(n-\frac{1}{2}\right)\Delta t} = \frac{\boldsymbol{E}^{S,n-1}+\boldsymbol{E}^{S,n}}{2} \tag{C.9}$$

$$\boldsymbol{E}^I\Big|_{t=\left(n-\frac{1}{2}\right)\Delta t} = \frac{\boldsymbol{E}^{I,n-1}+\boldsymbol{E}^{I,n}}{2} \tag{C.10}$$

Additionally, time-difference of electromagnetic fields were expressed as

$$\left.\frac{\partial \boldsymbol{E}^S}{\partial t}\right|_{t=\left(n-\frac{1}{2}\right)\Delta t} = \frac{\left(\boldsymbol{E}^{S,n}-\boldsymbol{E}^{S,n-1}\right)}{\Delta t} \tag{C.11}$$

$$\left.\frac{\partial \boldsymbol{E}^I}{\partial t}\right|_{t=\left(n-\frac{1}{2}\right)\Delta t} = \frac{\left(\boldsymbol{E}^{I,n}-\boldsymbol{E}^{I,n-1}\right)}{\Delta t} \tag{C.12}$$

$$\left.\frac{\partial \boldsymbol{H}^S}{\partial t}\right|_{t=n\Delta t} = \frac{\left(\boldsymbol{H}^{S,n}-\boldsymbol{H}^{S,n-1}\right)}{\Delta t} \tag{C.13}$$

$$\left.\frac{\partial \boldsymbol{H}^I}{\partial t}\right|_{t=n\Delta t} = \frac{\left(\boldsymbol{H}^{I,n}-\boldsymbol{H}^{I,n-1}\right)}{\Delta t} \tag{C.14}$$

By referring (C.9), (C.11) and (C.13), (C.7) and (C.8) are derived as

$$\begin{aligned}\boldsymbol{E}^{S,n} &= \frac{1-s\Delta t/2e}{1+s\Delta t/2e}\boldsymbol{E}^{S,n-1} + \frac{\Delta t/e}{1+s\Delta t/2e}\nabla\times\boldsymbol{H}^{S,n-\frac{1}{2}} - \frac{s\Delta t/e}{1+s\Delta t/2e}\boldsymbol{E}^I\Big|_{t=\left(n-\frac{1}{2}\right)\Delta t} \\ &\quad - \frac{(e-e_o)\Delta t/e}{1+s\Delta t/2e}\left.\frac{\partial \boldsymbol{E}^I}{\partial t}\right|_{t=\left(n-\frac{1}{2}\right)\Delta t}\end{aligned} \tag{C.15}$$

$$\boldsymbol{H}^{S,n+\frac{1}{2}} = \boldsymbol{H}^{S,n-\frac{1}{2}} - \frac{\Delta t}{m}\nabla\times\boldsymbol{E}^{S,n} - \left(\frac{m-m_o}{m}\right)\Delta t\left.\frac{\partial \boldsymbol{H}^I}{\partial t}\right|_{t=n\Delta t} \tag{C.16}$$

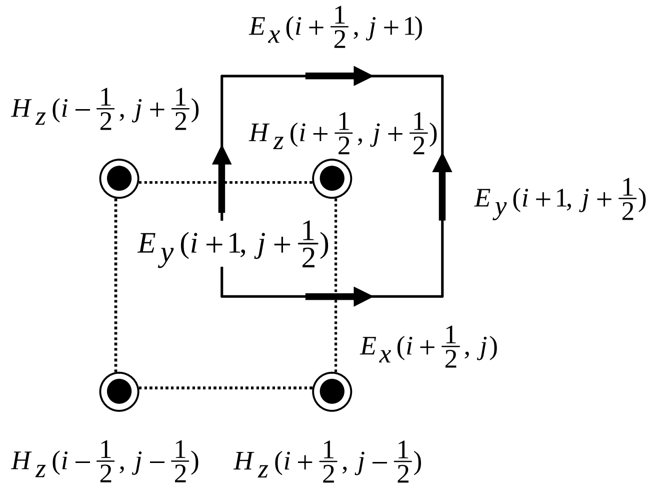

Figure C.1. Portion of the finite-difference grid.

In this study, horizontal polarisation or transverse electric (TE) mode is considered. Hence, electromagnetic field components are considered as $\boldsymbol{H}^S = \left(0,\ 0,\ H_z^S\right)$ and $\boldsymbol{E}^S = \left(E_x^S, E_y^S, 0\right)$. Then next relationships are obtained.

$$\nabla \times \boldsymbol{H}^S = \hat{x}\frac{\partial H_z^S}{\partial y} - \hat{y}\frac{\partial H_z^S}{\partial x} + \hat{z}0 \tag{C.17}$$

$$\nabla \times \boldsymbol{E}^S = -\hat{x}\frac{\partial E_y^S}{\partial z} + \hat{y}\frac{\partial E_x^S}{\partial z} + \hat{z}\left(\frac{\partial E_y^S}{\partial x} - \frac{\partial E_x^S}{\partial y}\right) \tag{C.18}$$

where these fields are considered in Cartesian-coordinate and $\hat{x}$, $\hat{y}$, $\hat{z}$ are unit vector of x, y, z axis. Then (C.17) and (C.18) are substituted into (C.15) and (C.16), hence each component of electromagnetic fields are obtained as

$$\begin{aligned} E_x^{S,n} = \frac{1 - s\Delta t/2e}{1 + s\Delta t/2e}E_x^{S,n-1} + \frac{\Delta t/e}{1 + s\Delta t/2e}\frac{\partial H_z^{S,n-\frac{1}{2}}}{\partial y} - \frac{s\Delta t/e}{1 + s\Delta t/2e}E_x^I\Big|_{t=\left(n-\frac{1}{2}\right)\Delta t} \\ - \frac{(e - e_o)\Delta t/e}{1 + s\Delta t/2e}\frac{\partial E_x^I}{\partial t}\Big|_{t=\left(n-\frac{1}{2}\right)\Delta t} \end{aligned} \tag{C.19}$$

$$
\begin{aligned}
E_y^{S,n} = {} & \frac{1-s\Delta t/2e}{1+s\Delta t/2e} E_y^{S,n-1} - \frac{\Delta t/e}{1+s\Delta t/2e} \frac{\partial H_z^{S,n-\frac{1}{2}}}{\partial x} - \frac{s\Delta t/e}{1+s\Delta t/2e} E_y^{I}\bigg|_{t=\left(n-\frac{1}{2}\right)\Delta t} \\
& - \frac{(e-e_o)\Delta t/e}{1+s\Delta t/2e} \frac{\partial E_y^{I}}{\partial t}\bigg|_{t=\left(n-\frac{1}{2}\right)\Delta t}
\end{aligned}
\qquad \text{(C.20)}
$$

$$
H_z^{S,n+\frac{1}{2}} = H_z^{S,n-\frac{1}{2}} - \frac{\Delta t}{m}\left(\frac{\partial E_y^{S,n}}{\partial x} - \frac{\partial E_x^{S,n}}{\partial y} \right) - \left(\frac{m-m_o}{m} \right)\Delta t \frac{\partial H_z^{I}}{\partial t}\bigg|_{t=n\Delta t} \qquad \text{(C.21)}
$$

By referring figure C.1, each electromagnetic field is obtained by substituting (C.10), (C.12), and (C.14) into (C.19) to (C.21) and by using Yee’s expression (Yee 1966), as

$$
\begin{aligned}
E_x^{S,n}\left(i+\tfrac{1}{2}, j\right) = {} & \frac{1-s\left(i+\frac{1}{2}, j\right)\Delta t/2e\left(i+\frac{1}{2}, j\right)}{1+s\left(i+\frac{1}{2}, j\right)\Delta t/2e\left(i+\frac{1}{2}, j\right)} E_x^{S,n-1}\left(i+\tfrac{1}{2}, j\right) \\
& + \frac{\Delta t/e\left(i+\frac{1}{2}, j\right)}{1+s\left(i+\frac{1}{2}, j\right)\Delta t/2e\left(i+\frac{1}{2}, j\right)} \frac{H_z^{S,n-\frac{1}{2}}\left(i+\frac{1}{2}, j+\frac{1}{2}\right) - H_z^{S,n-\frac{1}{2}}\left(i+\frac{1}{2}, j-\frac{1}{2}\right)}{\Delta y} \\
& - \frac{s\left(i+\frac{1}{2}, j\right)\Delta t/2e\left(i+\frac{1}{2}, j\right) - \left(e\left(i+\frac{1}{2}, j\right)-e_o\right)/e\left(i+\frac{1}{2}, j\right)}{1+s\left(i+\frac{1}{2}, j\right)\Delta t/2e\left(i+\frac{1}{2}, j\right)} E_x^{I,n-1}\left(i+\tfrac{1}{2}, j\right) \\
& - \frac{s\left(i+\frac{1}{2}, j\right)\Delta t/2e\left(i+\frac{1}{2}, j\right) + \left(e\left(i+\frac{1}{2}, j\right)-e_o\right)/e\left(i+\frac{1}{2}, j\right)}{1+s\left(i+\frac{1}{2}, j\right)\Delta t/2e\left(i+\frac{1}{2}, j\right)} E_x^{I,n}\left(i+\tfrac{1}{2}, j\right)
\end{aligned}
$$

(C.22)

$$
\begin{aligned}
E_y^{S,n}\left(i, j+\tfrac{1}{2}\right) = {} & \frac{1-s\left(i, j+\frac{1}{2}\right)\Delta t/2e\left(i, j+\frac{1}{2}\right)}{1+s\left(i, j+\frac{1}{2}\right)\Delta t/2e\left(i, j+\frac{1}{2}\right)} E_y^{S,n-1}\left(i, j+\tfrac{1}{2}\right) \\
& - \frac{\Delta t/e\left(i, j+\frac{1}{2}\right)}{1+s\left(i, j+\frac{1}{2}\right)\Delta t/2e\left(i, j+\frac{1}{2}\right)} \frac{H_z^{S,n-\frac{1}{2}}\left(i+\frac{1}{2}, j+\frac{1}{2}\right) - H_z^{S,n-\frac{1}{2}}\left(i-\frac{1}{2}, j+\frac{1}{2}\right)}{\Delta x} \\
& - \frac{s\left(i, j+\frac{1}{2}\right)\Delta t/2e\left(i, j+\frac{1}{2}\right) - \left(e\left(i, j+\frac{1}{2}\right)-e_o\right)/e\left(i, j+\frac{1}{2}\right)}{1+s\left(i, j+\frac{1}{2}\right)\Delta t/2e\left(i, j+\frac{1}{2}\right)} E_y^{I,n-1}\left(i, j+\tfrac{1}{2}\right) \\
& - \frac{s\left(i, j+\frac{1}{2}\right)\Delta t/2e\left(i, j+\frac{1}{2}\right) + \left(e\left(i, j+\frac{1}{2}\right)-e_o\right)/e\left(i, j+\frac{1}{2}\right)}{1+s\left(i, j+\frac{1}{2}\right)\Delta t/2e\left(i, j+\frac{1}{2}\right)} E_y^{I,n}\left(i, j+\tfrac{1}{2}\right)
\end{aligned}
$$

(C.23)

$$H_z^{S,n+\frac{1}{2}}\left(i+\tfrac{1}{2},\, j+\tfrac{1}{2}\right)=$$

$$H_z^{S,n-\frac{1}{2}}\left(i+\tfrac{1}{2},\, j+\tfrac{1}{2}\right)-\frac{\Delta t}{m\left(i+\frac{1}{2},\, j+\frac{1}{2}\right)\Delta x}\left(E_y^{S,n}\left(i+1,\, j+\tfrac{1}{2}\right)-E_y^{S,n}\left(i,\, j+\tfrac{1}{2}\right)\right)$$

$$+\frac{\Delta t}{m\left(i+\frac{1}{2},\, j+\frac{1}{2}\right)\Delta y}\left(E_x^{S,n}\left(i+\tfrac{1}{2},\, j+1\right)-E_x^{S,n}\left(i+\tfrac{1}{2},\, j\right)\right)$$

$$-\frac{m\left(i+\frac{1}{2},\, j+\frac{1}{2}\right)-m_o}{m\left(i+\frac{1}{2},\, j+\frac{1}{2}\right)}\left(H_z^{I,n+\frac{1}{2}}\left(i+\tfrac{1}{2},\, j+\tfrac{1}{2}\right)-H_z^{I,n-\frac{1}{2}}\left(i+\tfrac{1}{2},\, j+\tfrac{1}{2}\right)\right)$$

(C.24)

References

1. YEE, K.S., 1966, Numerical solution of initial boundary value problems involving Maxwell’s equations in isotropic media. IEEE Transactions on Antennas Propagation, 14, 302-307.